Lecture 11

2025/2026

Microwave Devices and Circuits for Radiocommunications

2025/2026

- 2C/1L, **MDCR**
- **Attendance at minimum 7 sessions (course or laboratory)**
- Lectures- **associate professor Radu Damian**
  - Tuesday **12-14**, **P2**
  - E – 50% final grade
  - **problems** + (2p atten. lect.) + (3 tests) + (bonus activity)
    - first test L1: 24.02.2026 (t2 and t3 not announced, lecture)
    - 3att.=+0.5p
  - **all materials/equipments authorized**

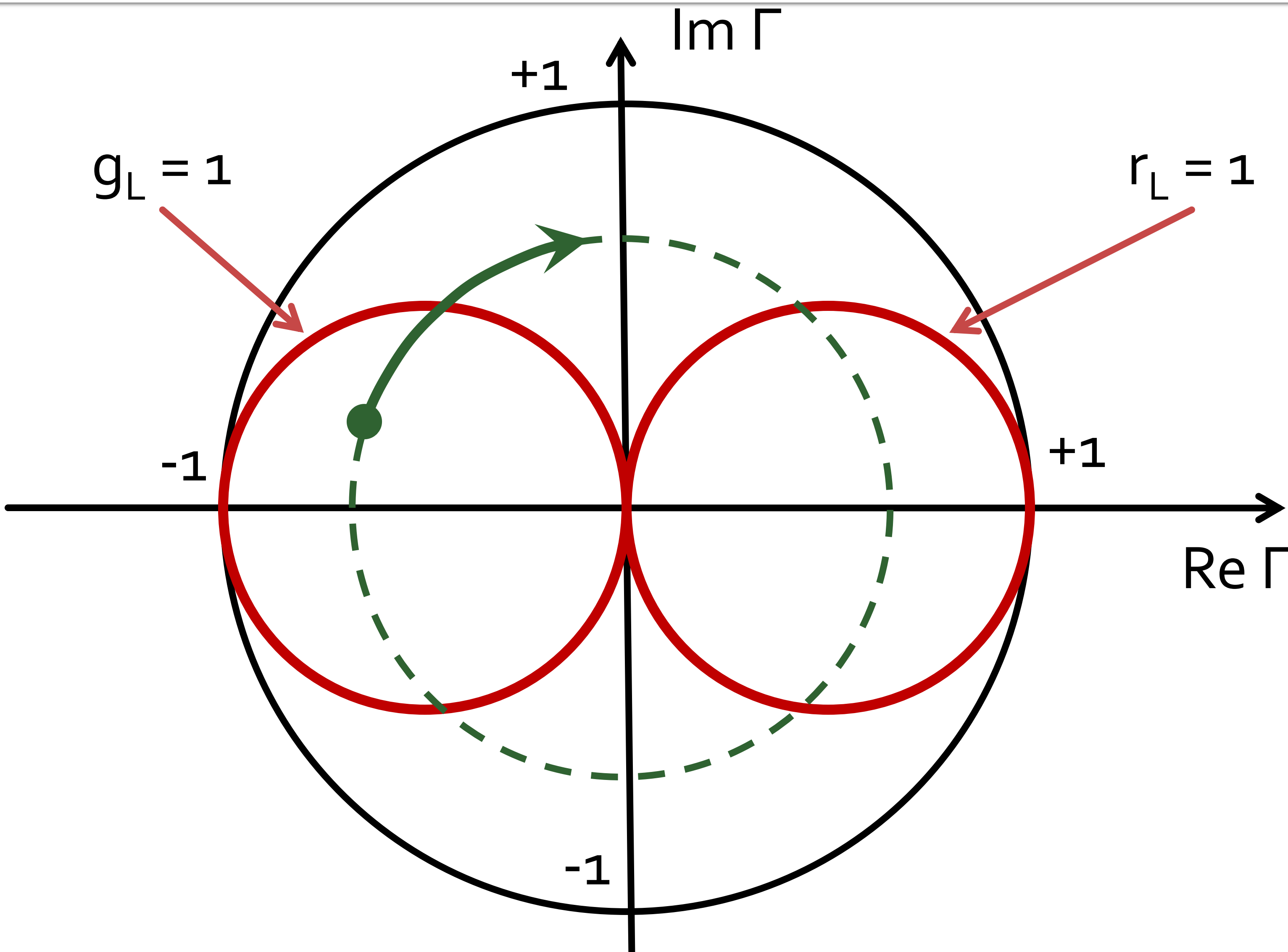
2025/2026

- Laboratory – **associate professor Radu Damian**
  - Monday 14-16, II.13 / (even weeks)
  - L – 25% final grade
    - ADS, 4 sessions
    - Attendance + **personal results**
  - P – 25% final grade
    - ADS, 3 sessions (-1? 24.02.2026)
    - personal homework

Impedance Matching

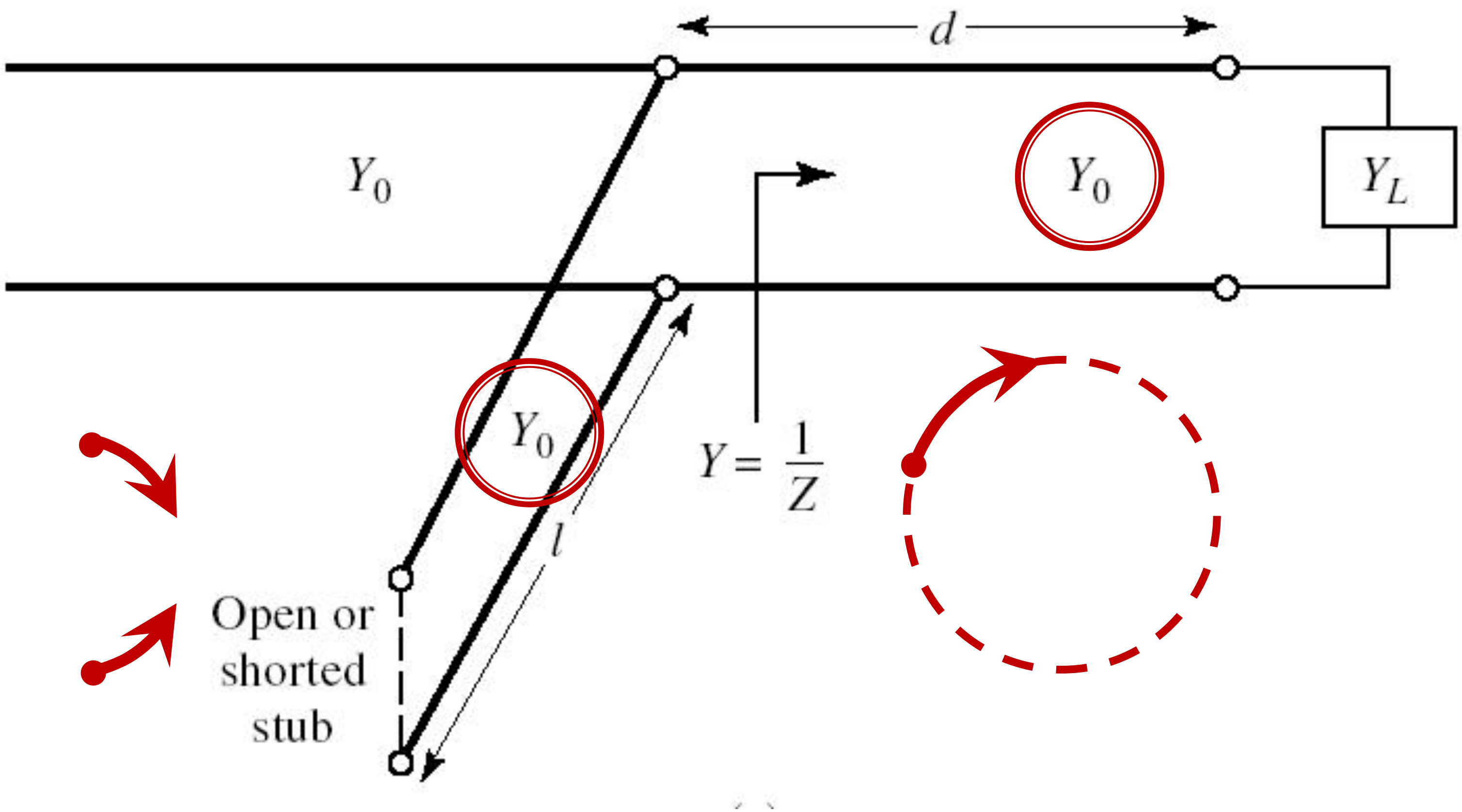
Impedance Matching with Stubs

Smith chart, r=1 and g=1



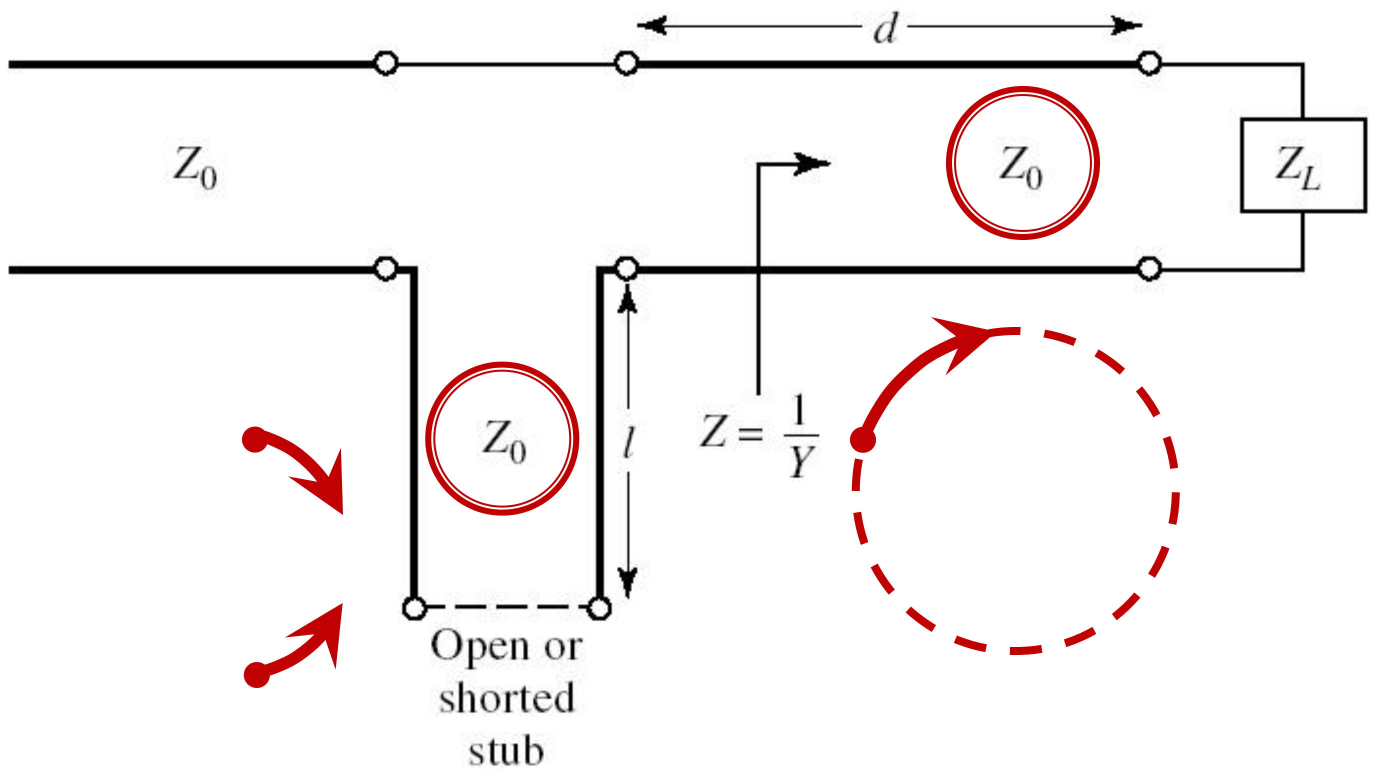
Single stub tuning

- Shunt Stub



Single stub tuning

- Series Stub
- difficult to realize in single conductor line technologies (microstrip)

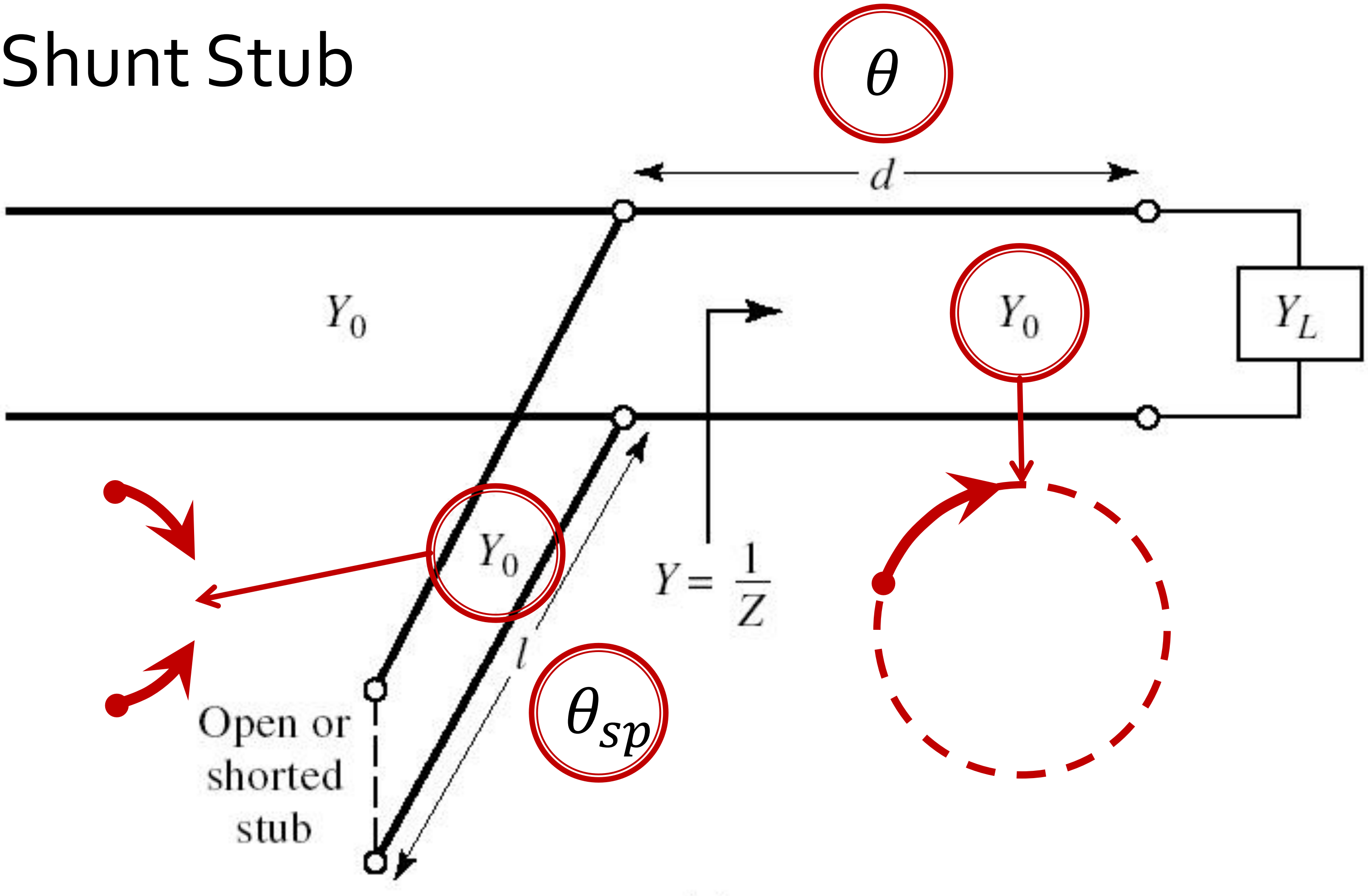


Analytical solutions

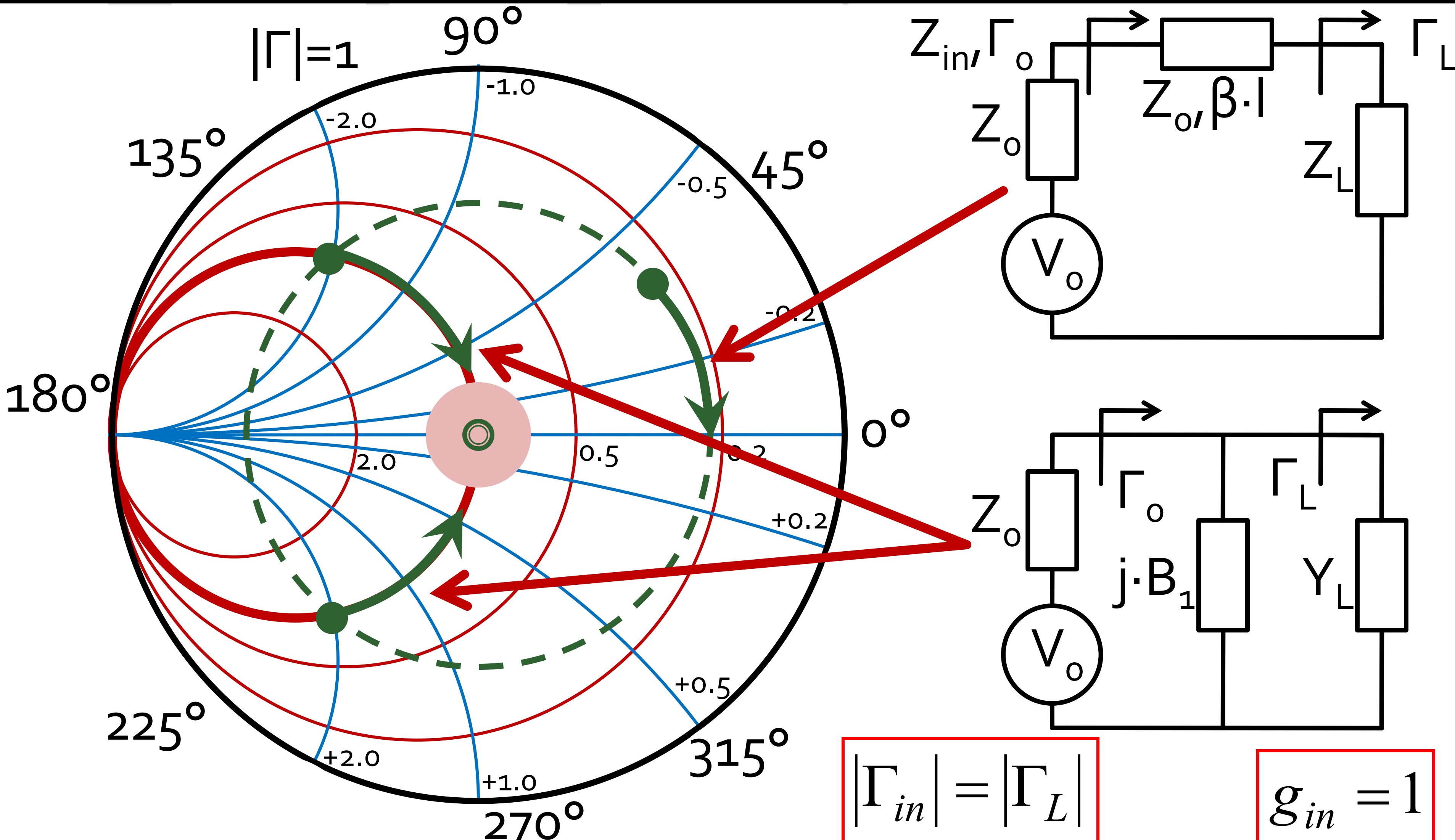
Exam / Project

Case 1, Shunt Stub

- Shunt Stub



Matching, series line + shunt susceptance



$$|\Gamma_{in}| = |\Gamma_L|$$

$$g_{in} = 1$$

Analytical solution, usage

$$\cos(\varphi + 2\theta) = -|\Gamma_S|$$

$$\theta_{sp} = \beta \cdot l = \tan^{-1} \frac{\mp 2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}}$$

$$\Gamma_S = 0.593\angle 46.85°$$

$$|\Gamma_S| = 0.593; \quad \varphi = 46.85° \qquad \cos(\varphi + 2\theta) = -0.593 \Rightarrow (\varphi + 2\theta) = \pm 126.35°$$

- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **shunt stub** equation
  - **"+" solution**

    $$(46.85° + 2\theta) = +126.35° \qquad \theta = +39.7° \qquad \operatorname{Im} y_S = \frac{-2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}} = -1.472$$

    $$\theta_{sp} = \tan^{-1}(\operatorname{Im} y_S) = -55.8°(+180°) \rightarrow \theta_{sp} = 124.2°$$

  - **"-" solution**

    $$(46.85° + 2\theta) = -126.35° \qquad \theta = -86.6°(+180°) \rightarrow \theta = 93.4°$$

    $$\operatorname{Im} y_S = \frac{+2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}} = +1.472 \qquad \theta_{sp} = \tan^{-1}(\operatorname{Im} y_S) = 55.8°$$

Analytical solution, usage

$$(\varphi+2\theta)=\begin{cases}+126.35^\circ\\-126.35^\circ\end{cases}\quad \theta=\begin{cases}39.7^\circ\\93.4^\circ\end{cases}\quad \mathrm{Im}[y_S(\theta)]=\begin{cases}-1.472\\+1.472\end{cases}\quad \theta_{sp}=\begin{cases}-55.8^\circ+180^\circ=124.2^\circ\\+55.8^\circ\end{cases}$$

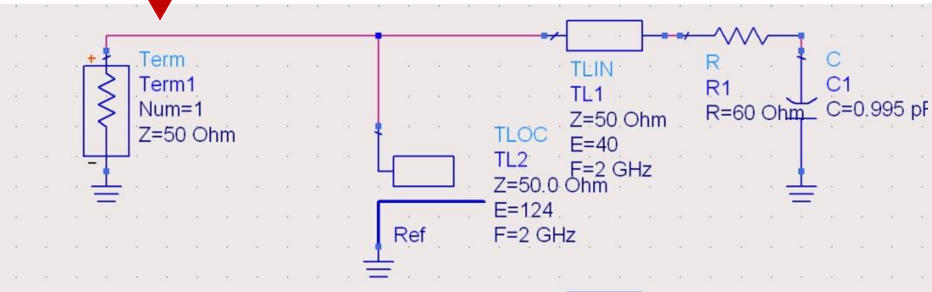
- We choose **one** of the two possible solutions
- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **shunt stub** equation

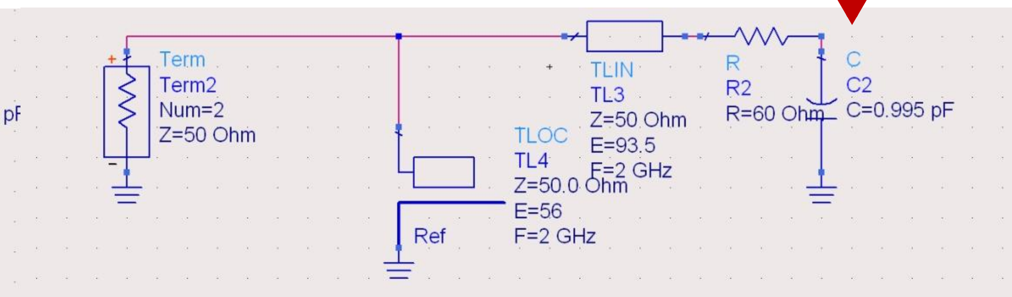
$$l_1=\frac{39.7^\circ}{360^\circ}\cdot\lambda=0.110\cdot\lambda$$

$$l_2=\frac{124.2^\circ}{360^\circ}\cdot\lambda=0.345\cdot\lambda$$

$$l_1=\frac{93.4^\circ}{360^\circ}\cdot\lambda=0.259\cdot\lambda$$

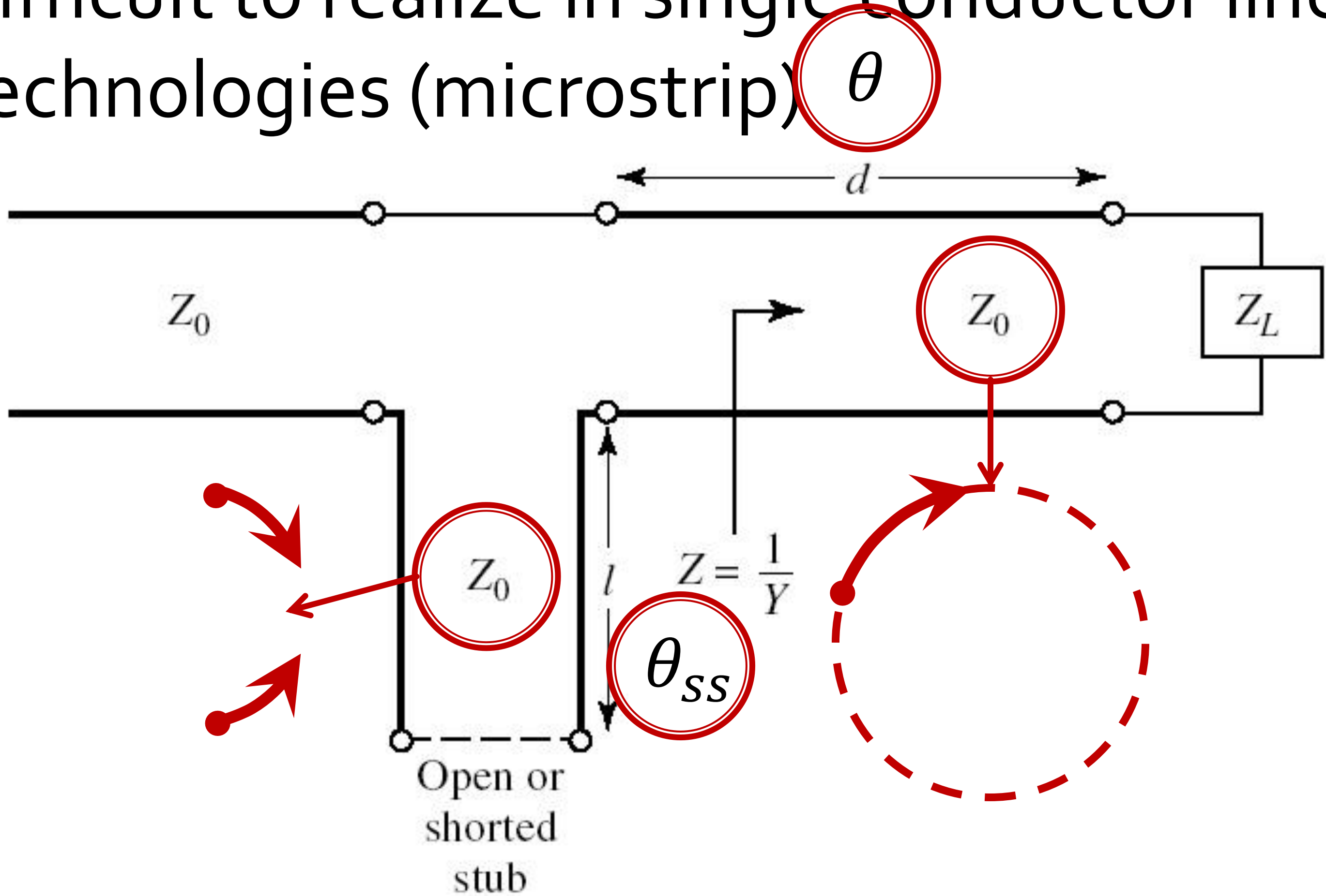
$$l_2=\frac{55.8^\circ}{360^\circ}\cdot\lambda=0.155\cdot\lambda$$



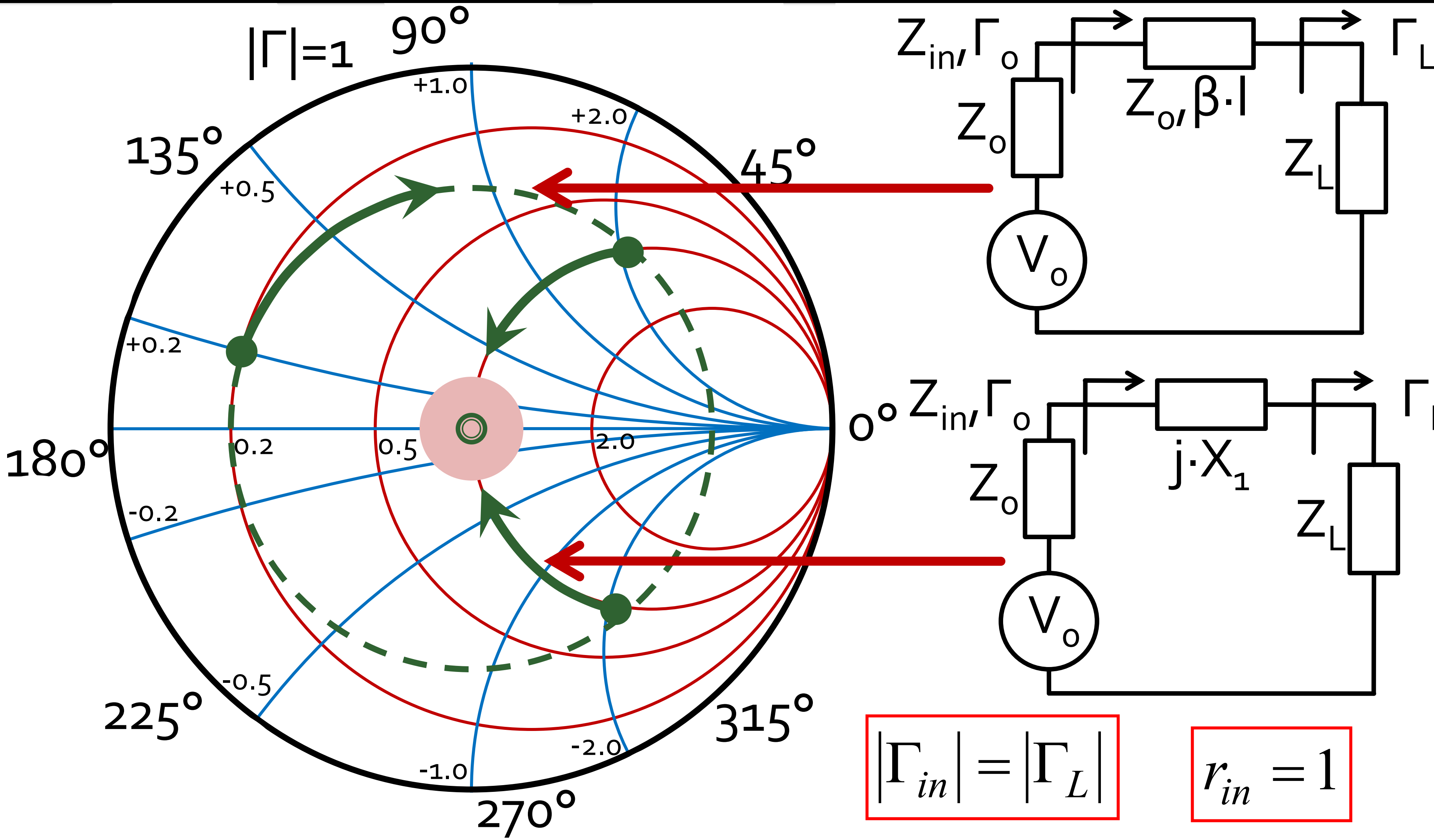


Case 2, Series Stub

- Series Stub
- difficult to realize in single conductor line technologies (microstrip)



Matching, series line + series reactance



$$|\Gamma_{in}| = |\Gamma_L|$$

$$r_{in} = 1$$

Analytical solution, usage

$$\cos(\varphi + 2\theta) = |\Gamma_S|$$

$$\theta_{ss} = \beta \cdot l = \cot^{-1} \frac{\mp 2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}}$$

$$\Gamma_S = 0.555 \angle -29.92^\circ$$

$$|\Gamma_S| = 0.555; \quad \varphi = -29.92^\circ \qquad \cos(\varphi + 2\theta) = 0.555 \Rightarrow (\varphi + 2\theta) = \pm 56.28^\circ$$

- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **series stub** equation
  - **"+" solution**

$$(-29.92^\circ + 2\theta) = +56.28^\circ \qquad \theta = 43.1^\circ \qquad \operatorname{Im} z_S = \frac{+2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}} = +1.335$$

$$\theta_{ss} = -\cot^{-1}(\operatorname{Im} z_S) = -36.8^\circ (+180^\circ) \rightarrow \theta_{ss} = 143.2^\circ$$

  - **"-" solution**

$$(-29.92^\circ + 2\theta) = -56.28^\circ \qquad \theta = -13.2^\circ (+180^\circ) \rightarrow \theta = 166.8^\circ$$

$$\operatorname{Im} z_S = \frac{-2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}} = -1.335 \qquad \theta_{ss} = -\cot^{-1}(\operatorname{Im} z_S) = 36.8^\circ$$

Analytical solution, usage

$$(\varphi+2\theta)=\begin{cases}+56.28^\circ\\-56.28^\circ\end{cases}\quad \theta=\begin{cases}43.1^\circ\\166.8^\circ\end{cases}\quad \mathrm{Im}[z_S(\theta)]=\begin{cases}+1.335\\-1.335\end{cases}\quad \theta_{ss}=\begin{cases}-36.8^\circ+180^\circ=143.2^\circ\\+36.8^\circ\end{cases}$$

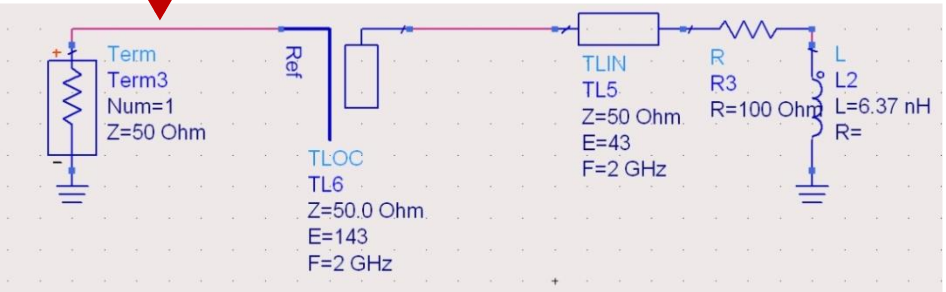
- We choose **one** of the two possible solutions
- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **series stub** equation

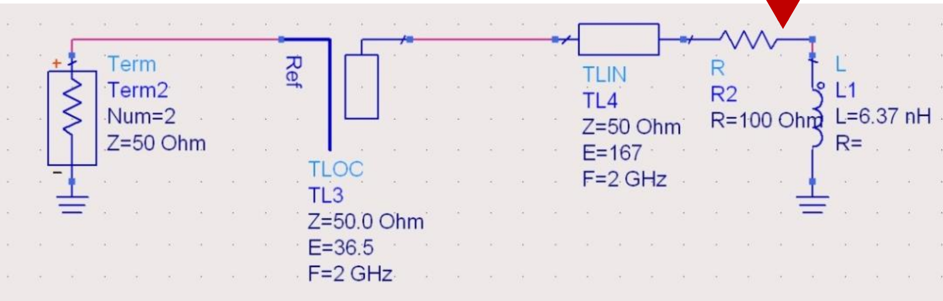
$$l_1=\frac{43.1^\circ}{360^\circ}\cdot\lambda=0.120\cdot\lambda$$

$$l_2=\frac{143.2^\circ}{360^\circ}\cdot\lambda=0.398\cdot\lambda$$

$$l_1=\frac{166.8^\circ}{360^\circ}\cdot\lambda=0.463\cdot\lambda$$

$$l_2=\frac{36.8^\circ}{360^\circ}\cdot\lambda=0.102\cdot\lambda$$





Stub, observations

- adding or subtracting **180°** (λ/2) doesn't change the result (full rotation around the Smith Chart)

$$E = \beta \cdot l = \pi = 180^\circ \qquad l = k \cdot \frac{\lambda}{2}, \forall k \in \mathbf{N}$$

  - if the lines/stubs result with **negative** "length"/ "electrical length" we add λ/2 / 180° to obtain physically realizable lines
- adding or subtracting **90°** (λ/4) change the stub impedance:

$$Z_{in,sc} = j \cdot Z_0 \cdot \tan \beta \cdot l \quad \Leftrightarrow \quad Z_{in,g} = -j \cdot Z_0 \cdot \cot \beta \cdot l$$

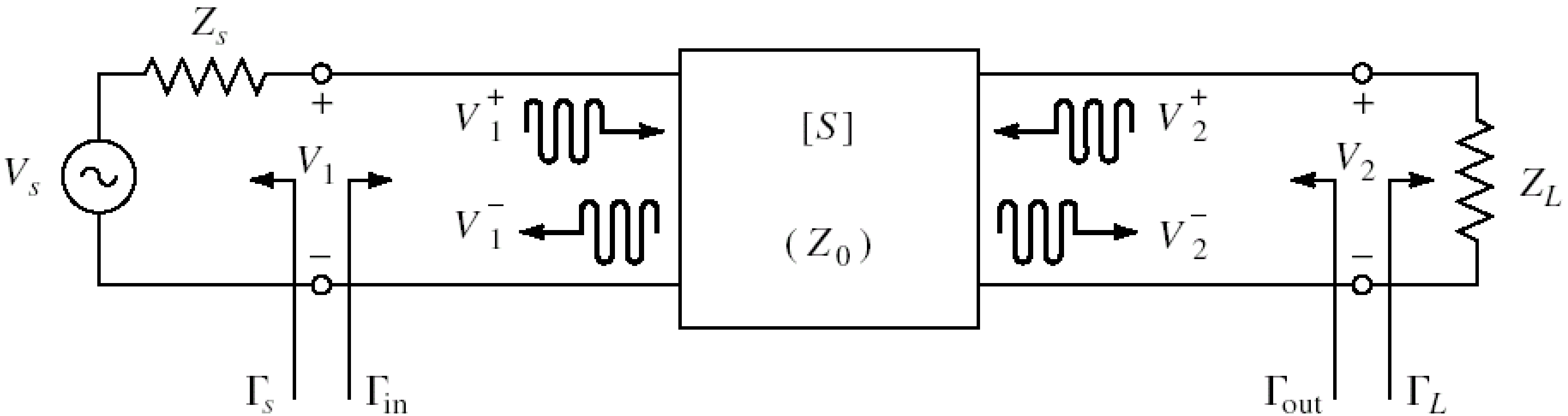
  - for the stub we can add or subtract 90° (λ/4) while in the same time changing **open-circuit** ⇔ **short-circuit**

Microwave Amplifiers

Course Topics

- Transmission lines
- Impedance matching and tuning
- Directional couplers
- Power dividers
- Microwave amplifier design
- Microwave filters
- ~~Oscillators and mixers~~ ?

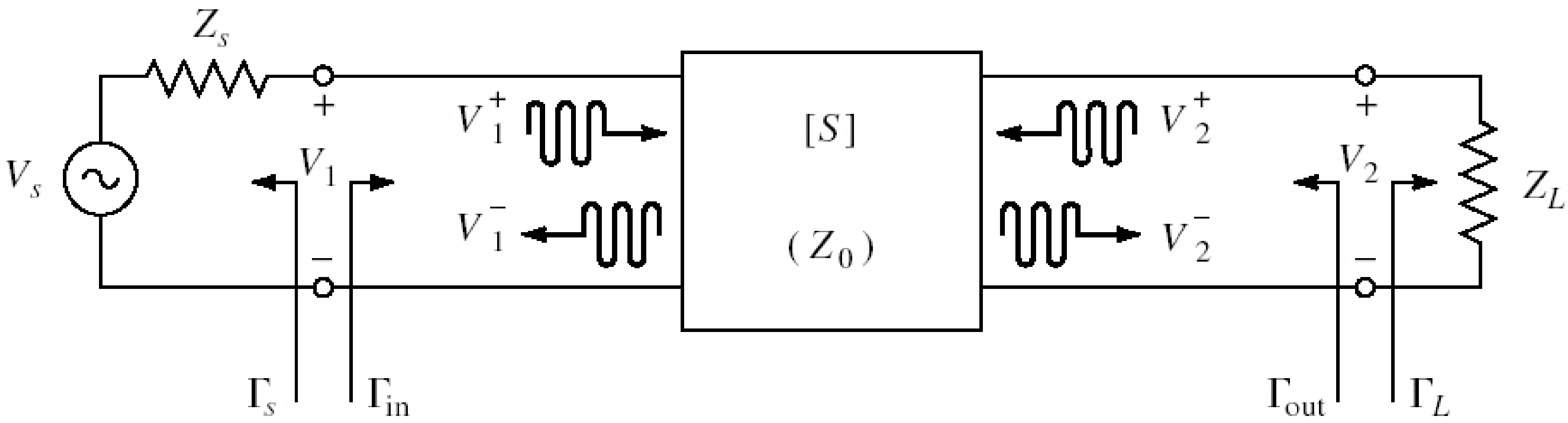
Amplifier as two-port



$$\Gamma_{in} = \frac{V_1^-}{V_1^+} = S_{11} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_L}{1 - S_{22} \cdot \Gamma_L}$$

$$\Gamma_{out} = \frac{V_2^-}{V_2^+} = S_{22} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_S}{1 - S_{11} \cdot \Gamma_S}$$

Amplifier as two-port



- For an amplifier two-port we are interested in:
  - **stability**
  - power gain
  - noise (sometimes – small signals)
  - linearity (sometimes – large signals)

Output stability circle (CSOUT)

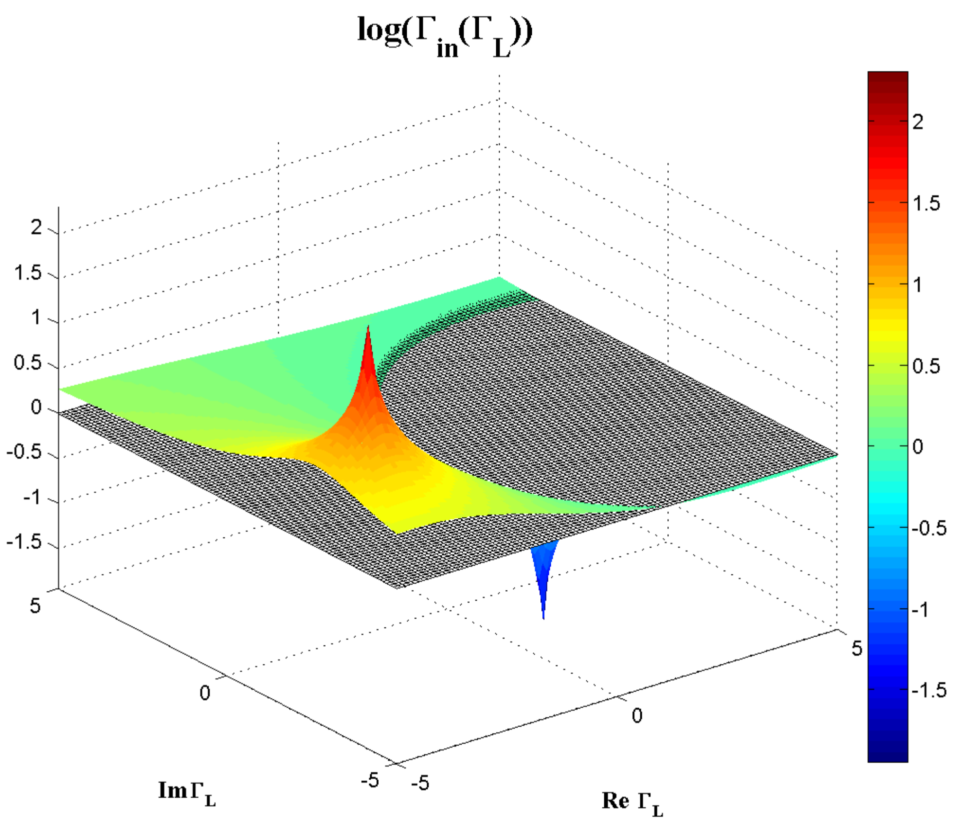
$$\left|\Gamma_L - \frac{\left(S_{22} - \Delta \cdot S_{11}^*\right)^*}{\left|S_{22}\right|^2 - \left|\Delta\right|^2}\right| = \left|\frac{S_{12} \cdot S_{21}}{\left|S_{22}\right|^2 - \left|\Delta\right|^2}\right| \qquad \left|\Gamma_L - C_L\right| = R_L$$

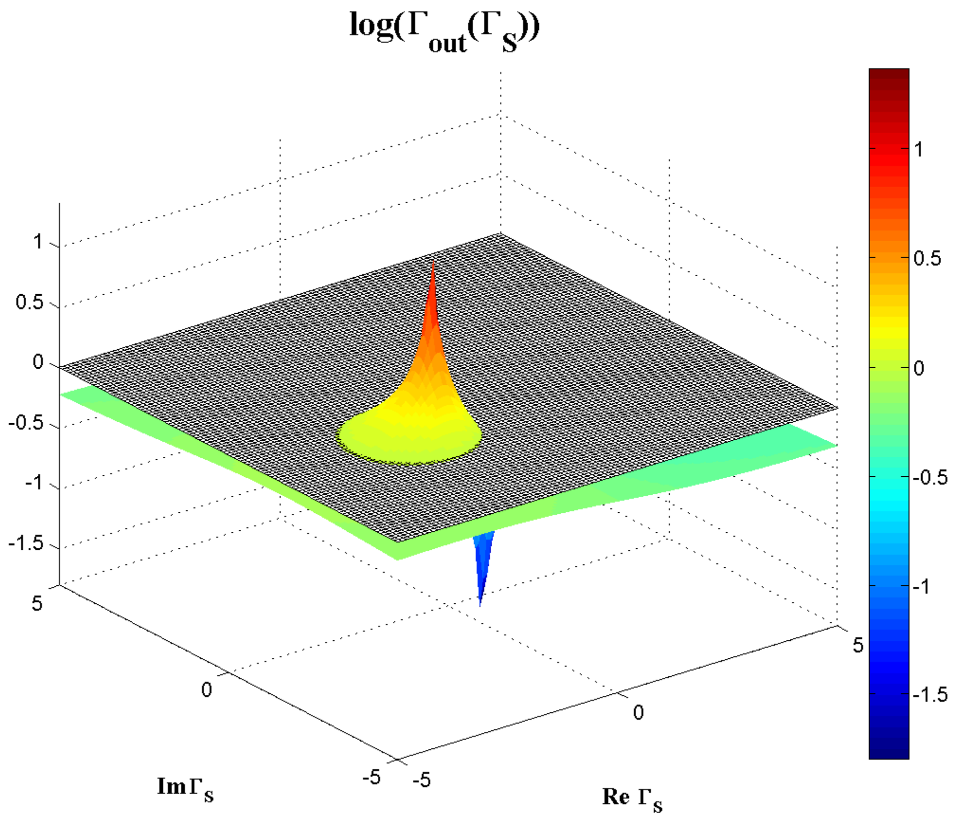
- We obtain the equation of a circle in the complex plane, which represents the locus of Γ_L for the **limit between stability and instability** ($|\Gamma_{in}| = 1$)
- This circle is the **output stability circle** (Γ_L)

$$C_L = \frac{\left(S_{22} - \Delta \cdot S_{11}^*\right)^*}{\left|S_{22}\right|^2 - \left|\Delta\right|^2} \qquad R_L = \frac{\left|S_{12} \cdot S_{21}\right|}{\left|\left|S_{22}\right|^2 - \left|\Delta\right|^2\right|}$$

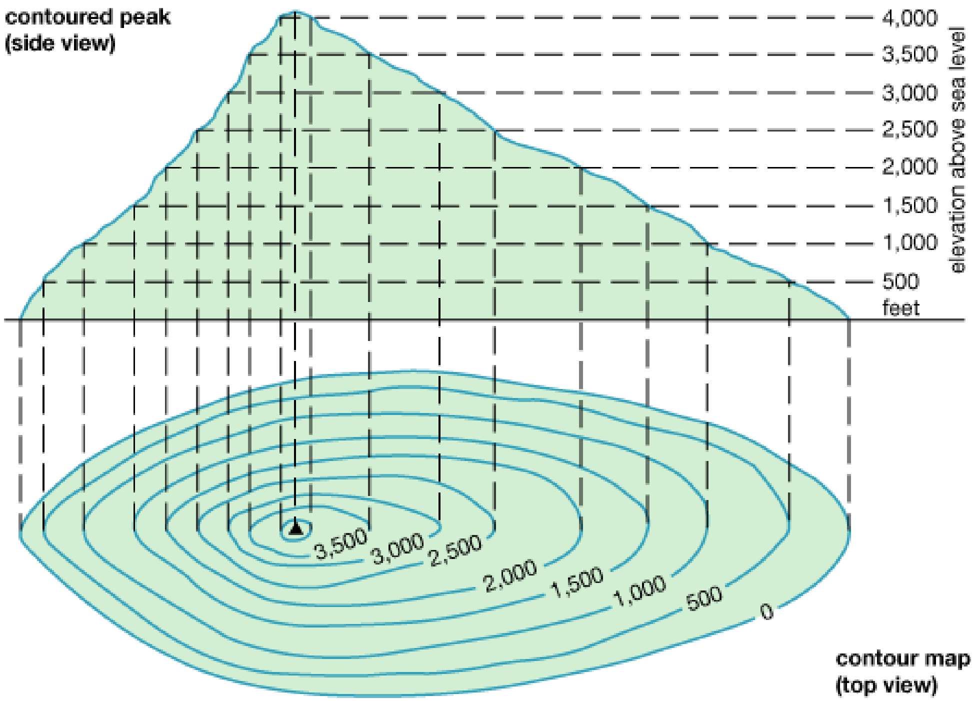
3D representation of $|\Gamma_{in}|$, $|\Gamma_{out}|$, $|\Gamma|=1$

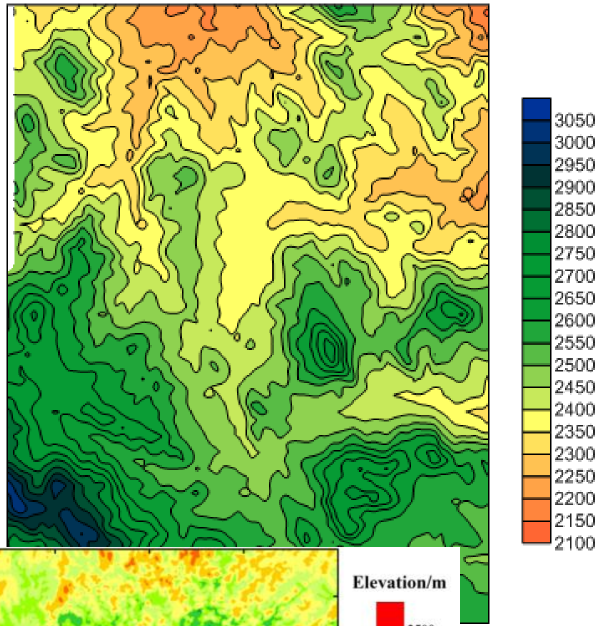
- $|\Gamma| = 1 \rightarrow \log_{10}|\Gamma| = 0$, the intersection with the plane z = 0 is a circle

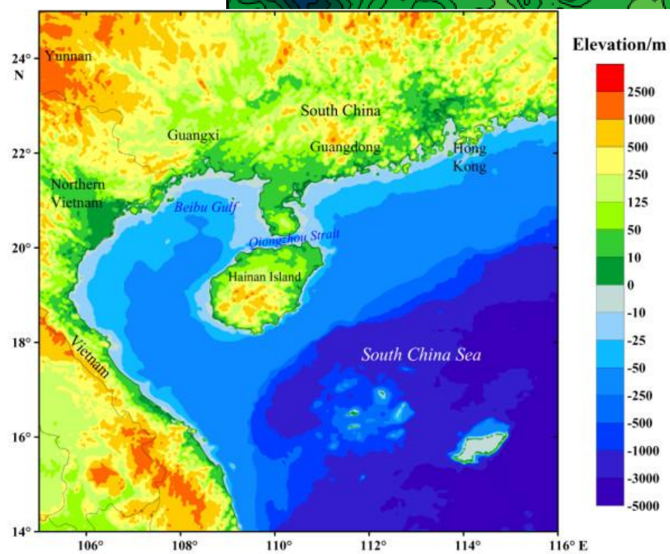




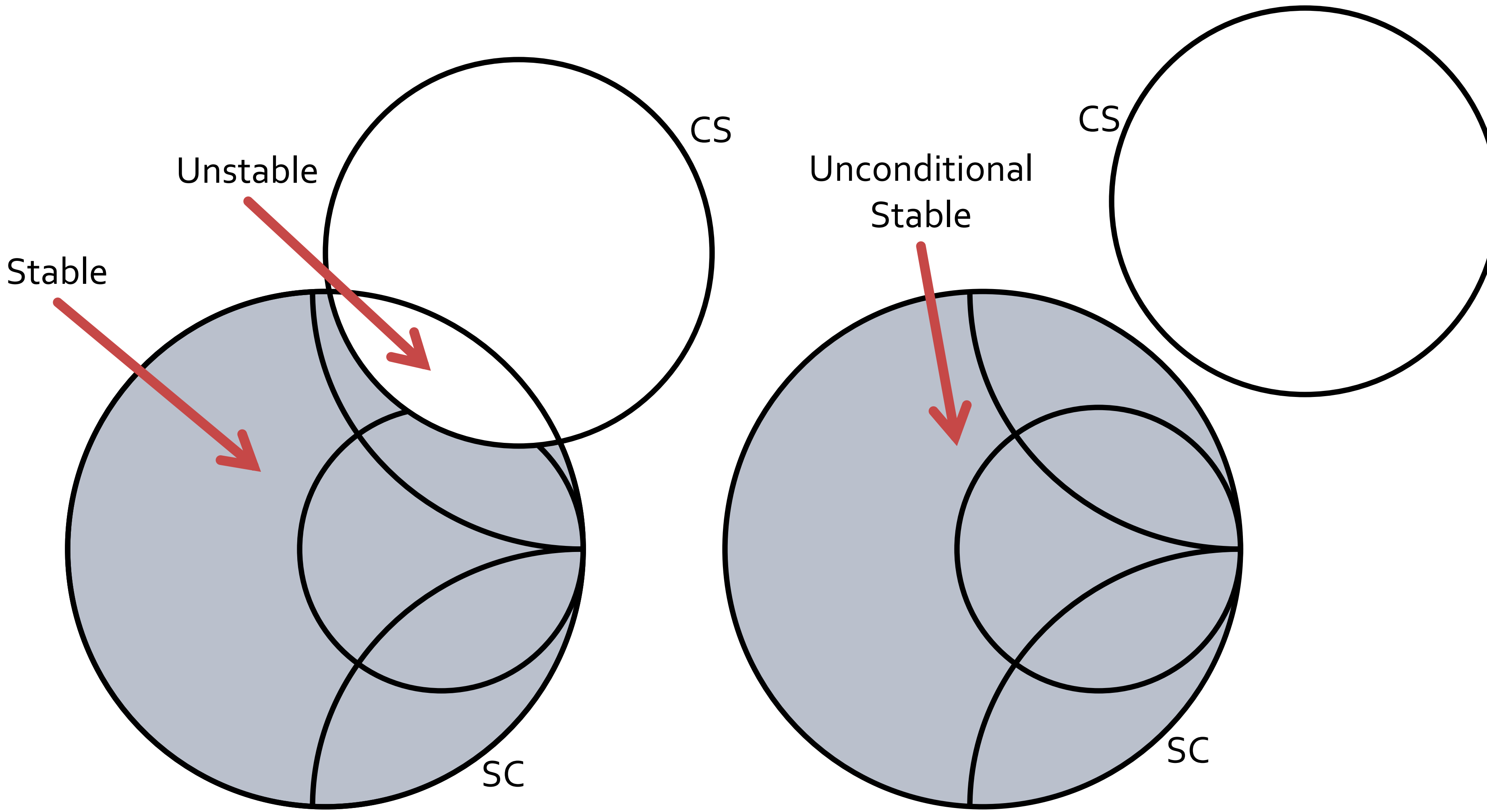
Contour map/lines



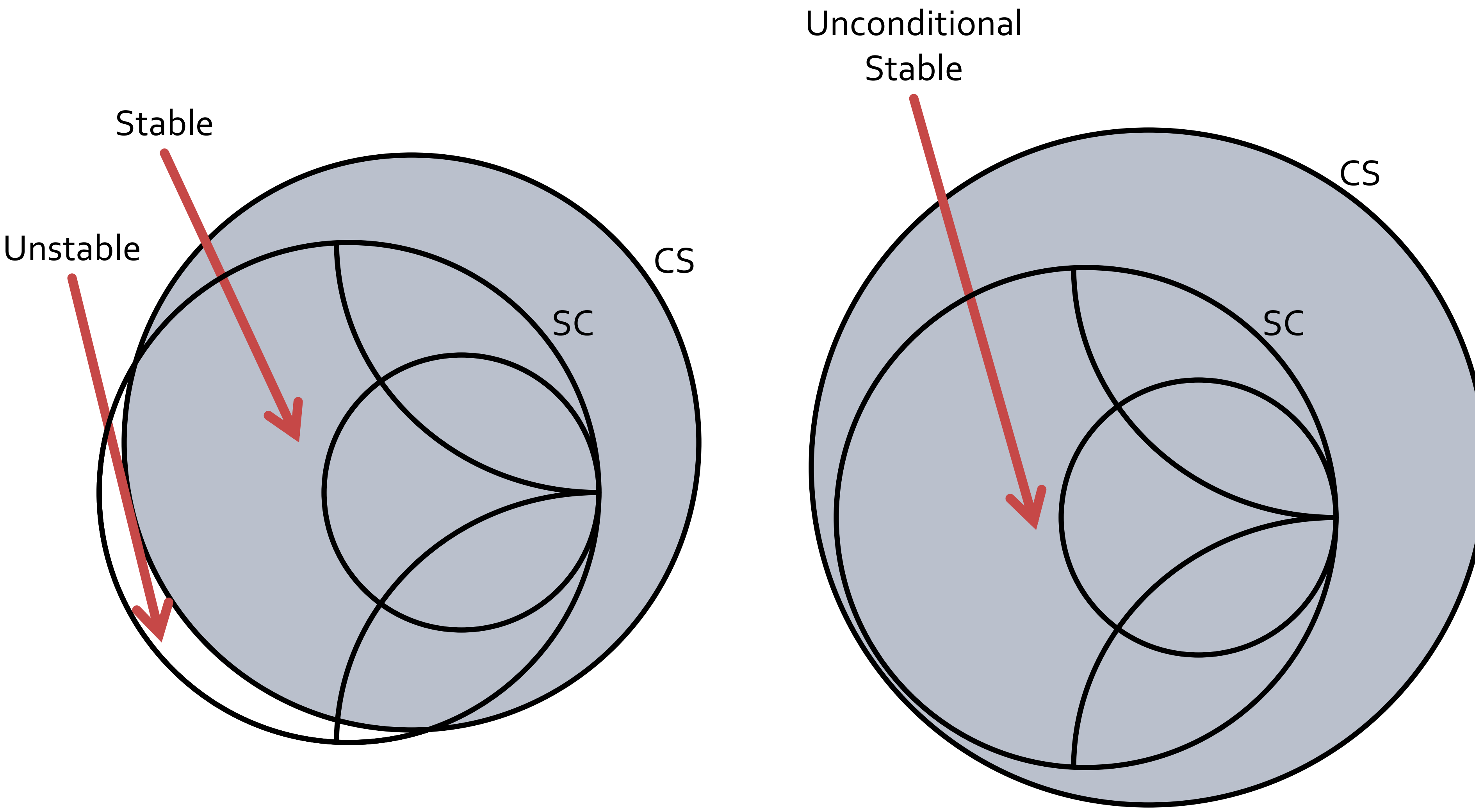




Several possible positioning

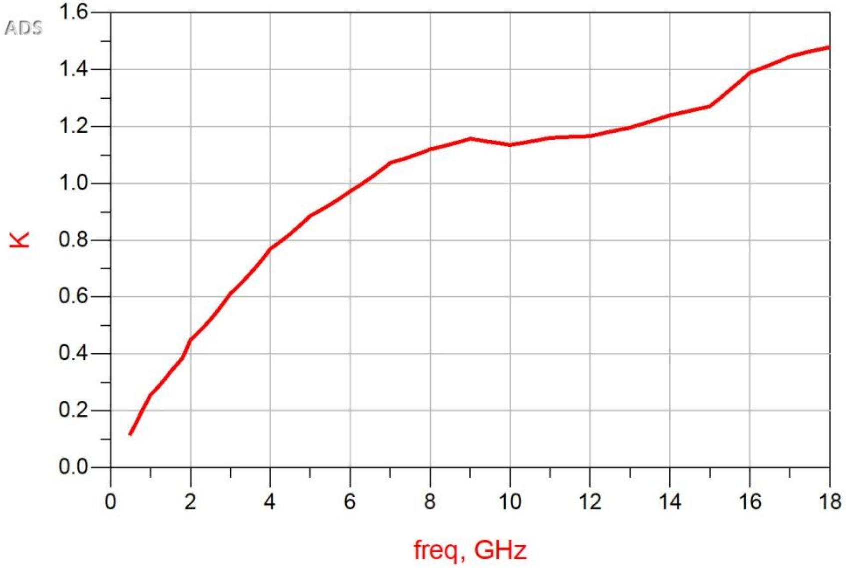


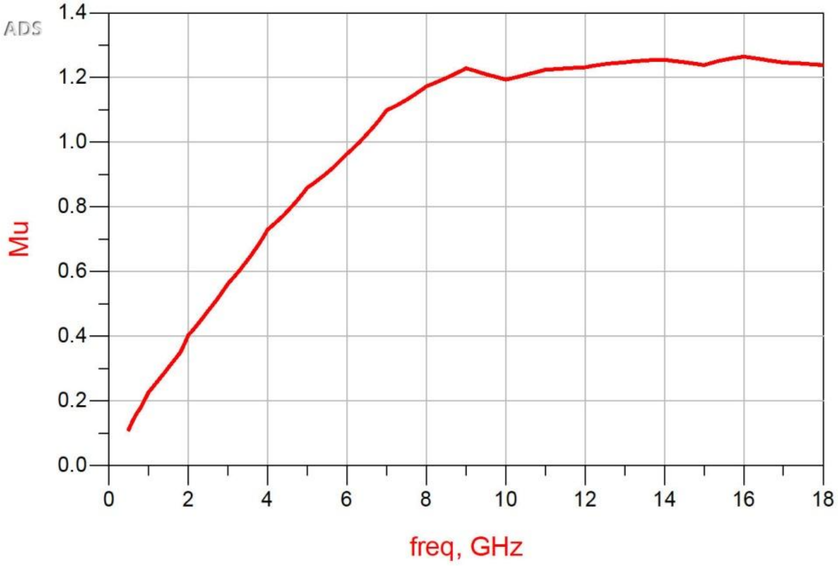
Several possible positioning



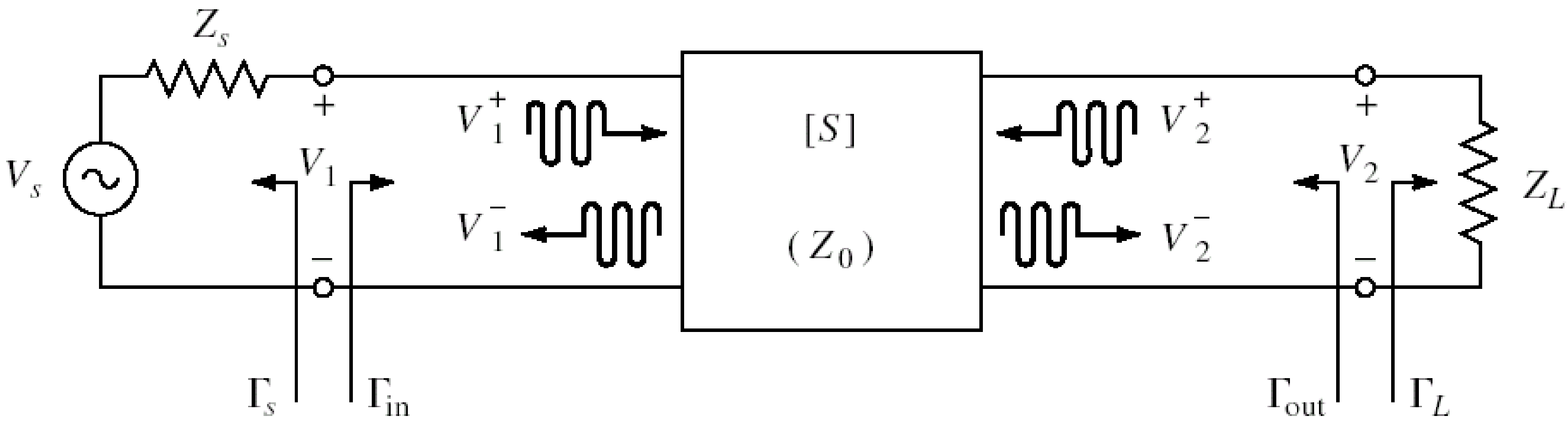
Stability

- ATF-34143 **at Vds=3V Id=20mA**.
- @0.5÷18GHz
- unconditionally stable for f > 6.31GHz





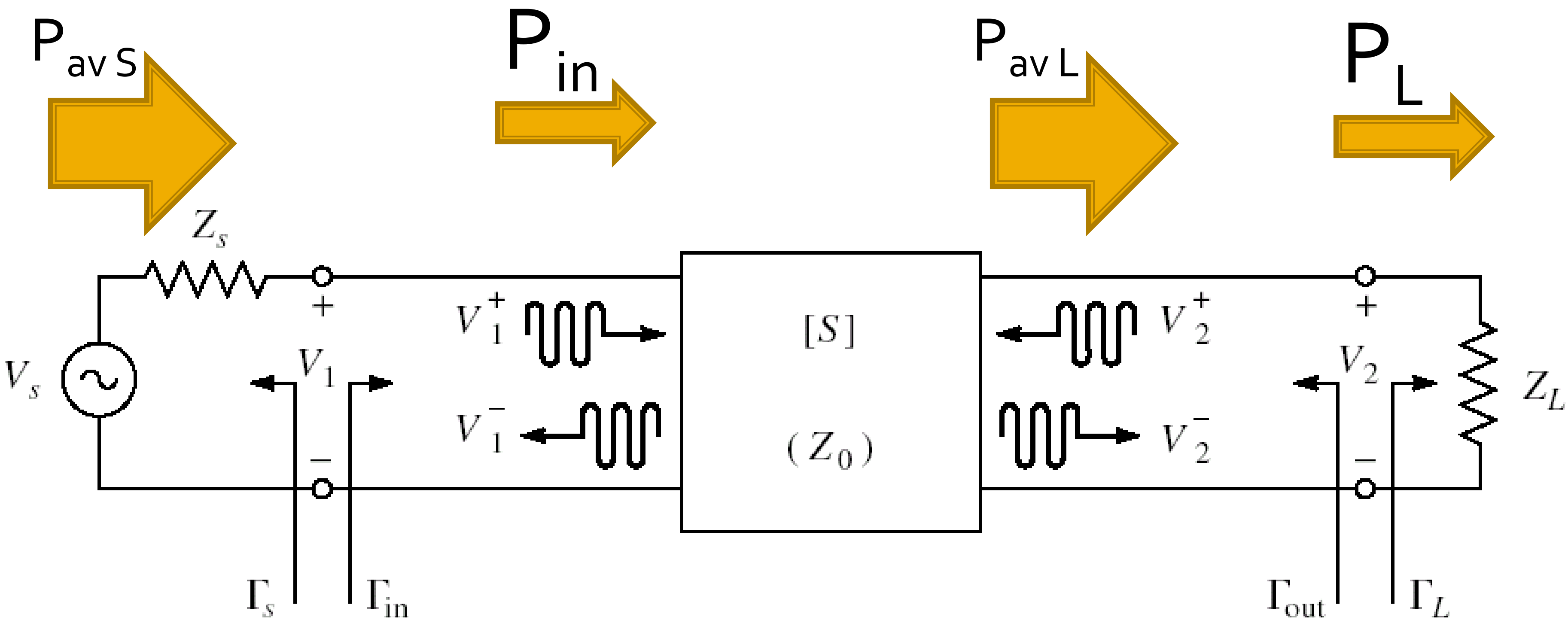
Amplifier as two-port



- For an amplifier two-port we are interested in:
  - stability
  - **power gain**
  - noise (sometimes – small signals)
  - linearity (sometimes – large signals)

Power / Matching

- Two ports in which matching influences the power transfer



Two-Port Power Gains

- **Available** power gain

$$G_A = \frac{P_{av\,L}}{P_{av\,S}} = \frac{|S_{21}|^2 \cdot \left(1-|\Gamma_S|^2\right)}{|1-S_{22}\cdot\Gamma_L|^2 \cdot \left(1-|\Gamma_{out}|^2\right)}$$

- **Transducer** power gain

$$G_T = \frac{P_L}{P_{av\,S}} = \frac{|S_{21}|^2 \cdot \left(1-|\Gamma_S|^2\right)\cdot\left(1-|\Gamma_L|^2\right)}{|1-\Gamma_S\cdot\Gamma_{in}|^2 \cdot |1-S_{22}\cdot\Gamma_L|^2}$$

$$\Gamma_{in} = \Gamma_{in}(\Gamma_L)$$

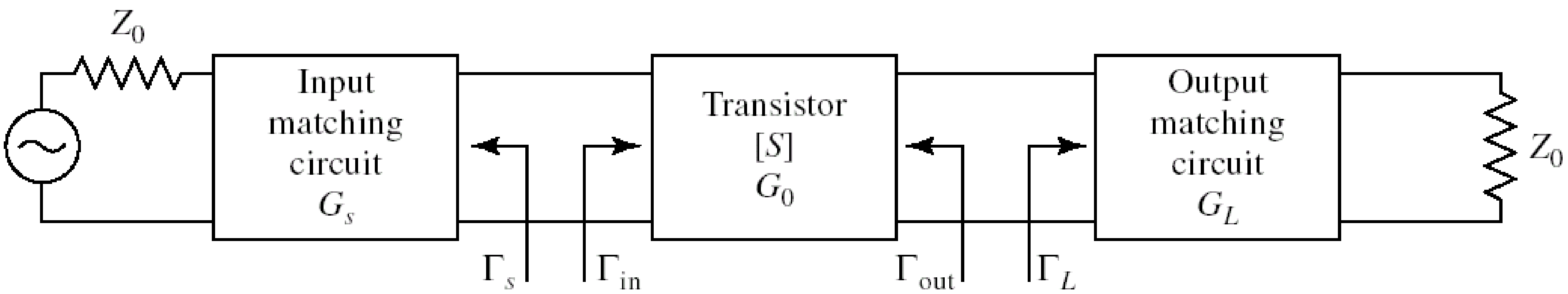
- **Unilateral transducer** power gain

$$G_{TU} = |S_{21}|^2 \cdot \frac{1-|\Gamma_S|^2}{|1-S_{11}\cdot\Gamma_S|^2} \cdot \frac{1-|\Gamma_L|^2}{|1-S_{22}\cdot\Gamma_L|^2}$$

$$S_{12} \cong 0 \qquad \Gamma_{in} = S_{11}$$

Input and output can be treated independently

Design for Maximum Gain



- Maximum power gain (complex conjugate matching):

$$\Gamma_{in} = \Gamma_S^* \qquad \Gamma_{out} = \Gamma_L^*$$

- For lossless matching sections

$$G_{T\max} = \frac{|S_{21}|^2 \cdot \left(1-|\Gamma_S|^2\right) \cdot \left(1-|\Gamma_L|^2\right)}{|1-\Gamma_S \cdot \Gamma_{in}|^2 \cdot |1-S_{22} \cdot \Gamma_L|^2} \qquad G_{T\max} = \frac{1}{1-|\Gamma_S|^2} \cdot |S_{21}|^2 \cdot \frac{1-|\Gamma_L|^2}{|1-S_{22} \cdot \Gamma_L|^2}$$

- For the general case of the bilateral transistor ($S_{12} \neq 0$) Γ_{in} and Γ_{out} depend on each other so the input and output sections must be matched simultaneously

Simultaneous matching

- Simultaneous matching can be achieved **if and only if** the amplifier is **unconditionally stable** at the operating frequency, and |Γ|<1 solutions are those with "–" sign of quadratic solutions

$$\Gamma_S = \frac{B_1 - \sqrt{B_1^2 - 4\cdot|C_1|^2}}{2\cdot C_1} \qquad \Gamma_L = \frac{B_2 - \sqrt{B_2^2 - 4\cdot|C_2|^2}}{2\cdot C_2}$$

$$\begin{cases} B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2 \\ C_1 = S_{11} - \Delta\cdot S_{22}^* \end{cases} \qquad \begin{cases} B_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta|^2 \\ C_2 = S_{22} - \Delta\cdot S_{11}^* \end{cases}$$

Design for Specified Gain

- Assumes the amplifier device **unilateral**

Input and output can be treated independently

$$G_{TU} = |S_{21}|^2 \cdot \frac{1-|\Gamma_S|^2}{|1-S_{11}\cdot\Gamma_S|^2} \cdot \frac{1-|\Gamma_L|^2}{|1-S_{22}\cdot\Gamma_L|^2}$$

$$S_{12} \cong 0$$

$$\Gamma_{in} = S_{11}$$

- Maximum power gain

$$\Gamma_S = S_{11}^*$$

$$\Gamma_L = S_{22}^*$$

$$G_{TU\max} = \frac{1}{1-|S_{11}|^2} \cdot |S_{21}|^2 \cdot \frac{1}{1-|S_{22}|^2}$$

Unilateral figure of merit

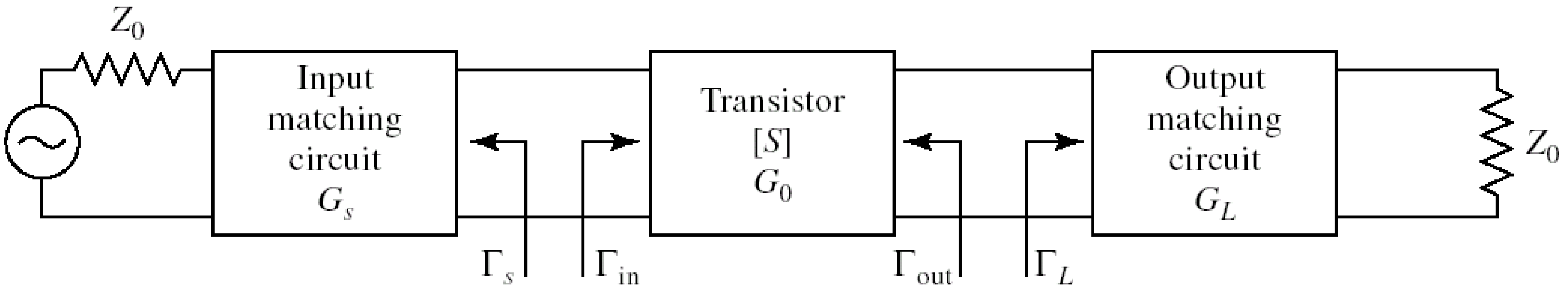
- Allows estimation of the error introduced by the unilateral assumption

$$\frac{1}{(1+U)^2} < \frac{G_T}{G_{TU}} < \frac{1}{(1-U)^2} \qquad U = \frac{|S_{12}|\cdot|S_{21}|\cdot|S_{11}|\cdot|S_{22}|}{\left(1-|S_{11}|^2\right)\cdot\left(1-|S_{22}|^2\right)}$$

- We compute U then the maximum and minimum deviation of G_{TU} from G_T
  - this deviation must be accounted in the design as a reserve gain against the target gain

$$-20\cdot\log(1+U) < G_T[dB] - G_{TU}[dB] < -20\cdot\log(1-U)$$

Design for Specified Gain



- In the unilateral assumption:

$$G_{TU} = \frac{1-|\Gamma_S|^2}{|1-S_{11}\cdot\Gamma_S|^2}\cdot|S_{21}|^2\cdot\frac{1-|\Gamma_L|^2}{|1-S_{22}\cdot\Gamma_L|^2}$$

$$G_S = \frac{1-|\Gamma_S|^2}{|1-S_{11}\cdot\Gamma_S|^2}$$

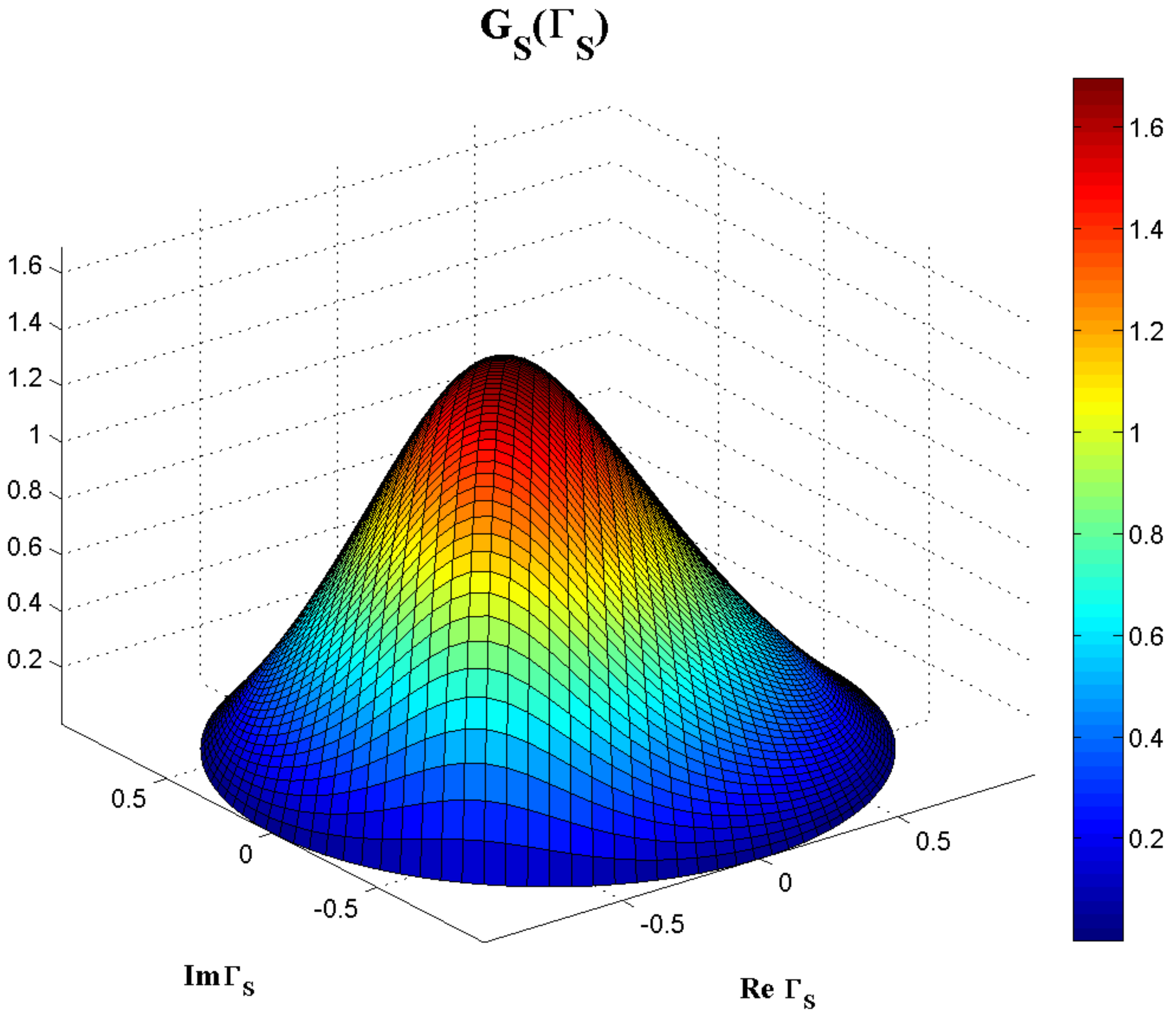
$$G_S = G_S(\Gamma_S)$$

$$G_0 = |S_{21}|^2$$

$$G_L = \frac{1-|\Gamma_L|^2}{|1-S_{22}\cdot\Gamma_L|^2}$$

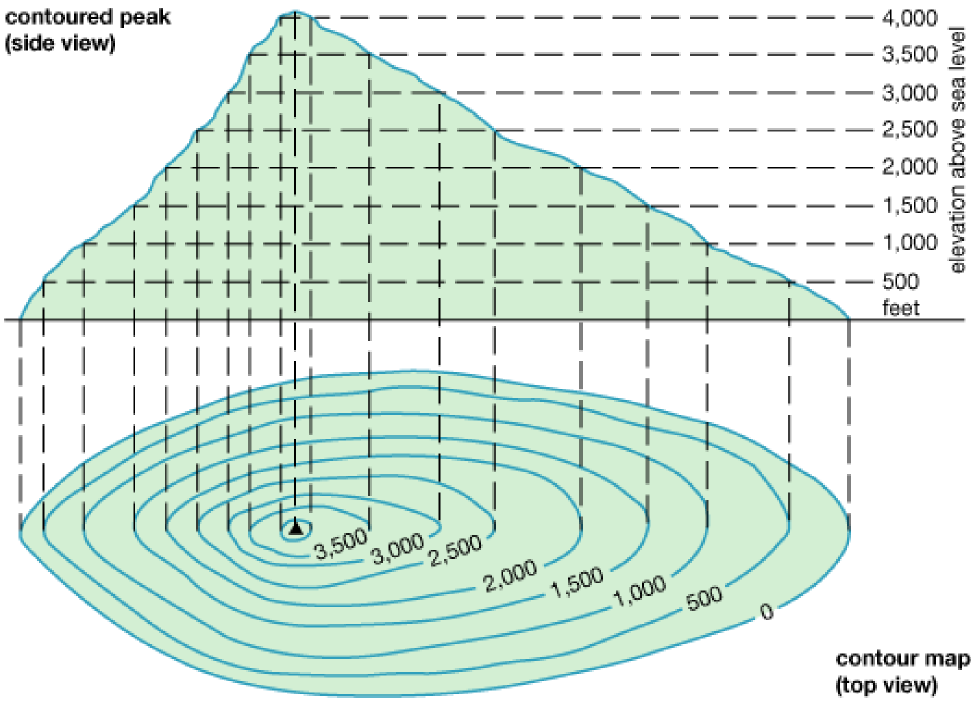
$$G_L = G_L(\Gamma_L)$$

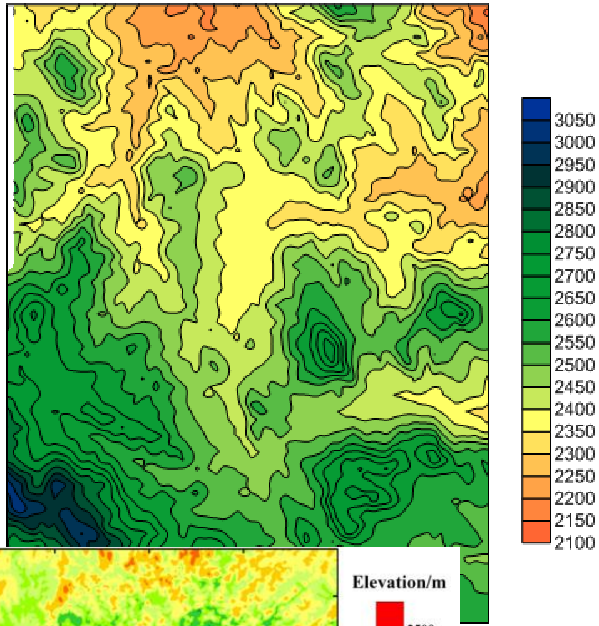
$G_S(\Gamma_S)$

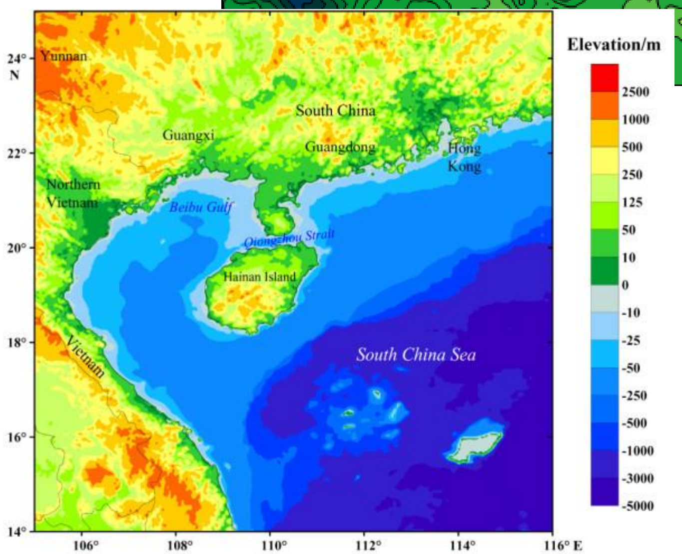


$$G_S = \frac{1 - |\Gamma_S|^2}{|1 - S_{11} \cdot \Gamma_S|^2}$$

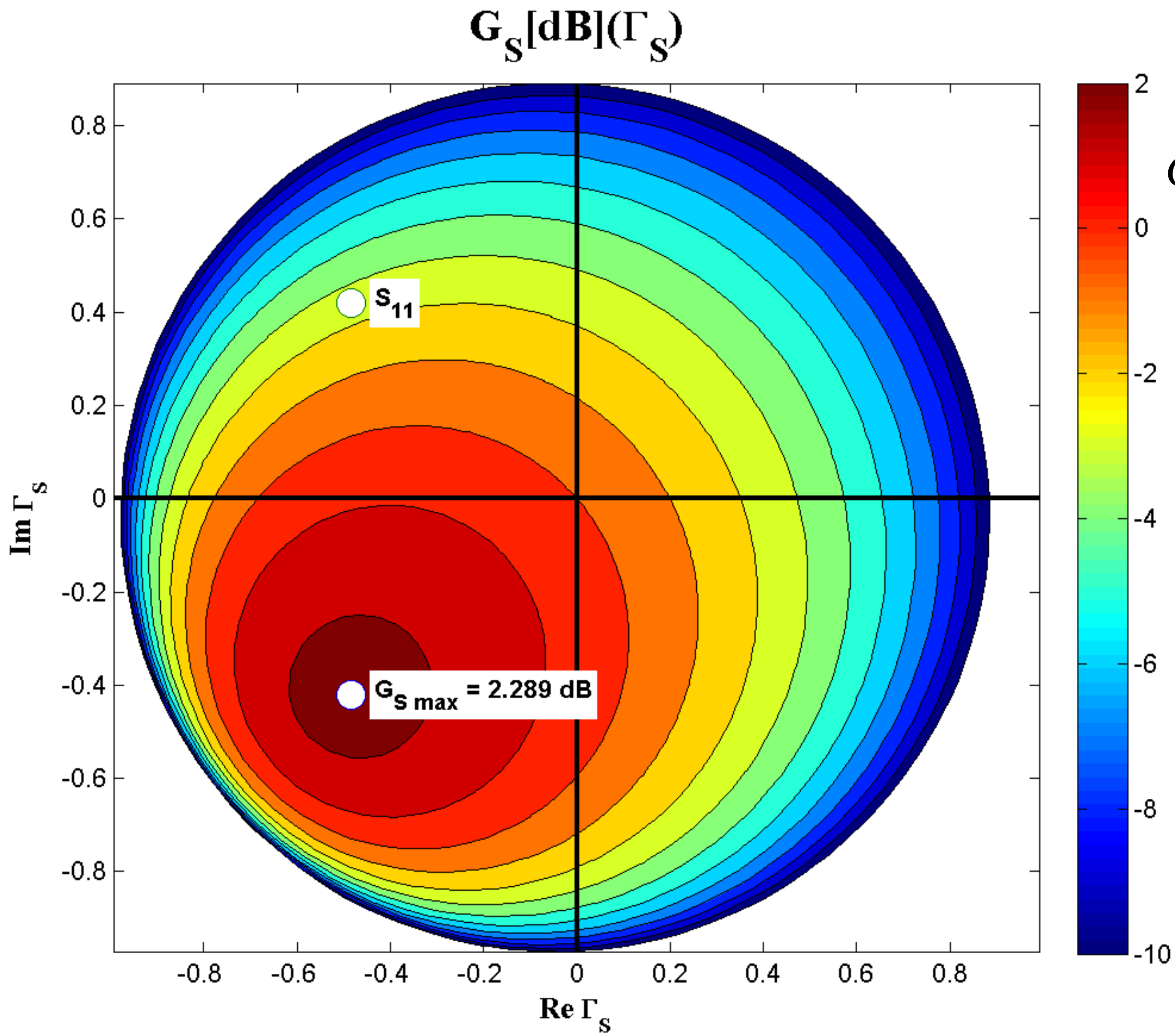
Contour map/lines







G_S[dB](Γ_S), constant value contours



$$G_S[dB] = 10 \cdot \log\left(\frac{1-|\Gamma_S|^2}{|1-S_{11}\cdot\Gamma_S|^2}\right)$$

$$G_{S\max} = G_S\Big|_{\Gamma_S = S_{11}^*}$$

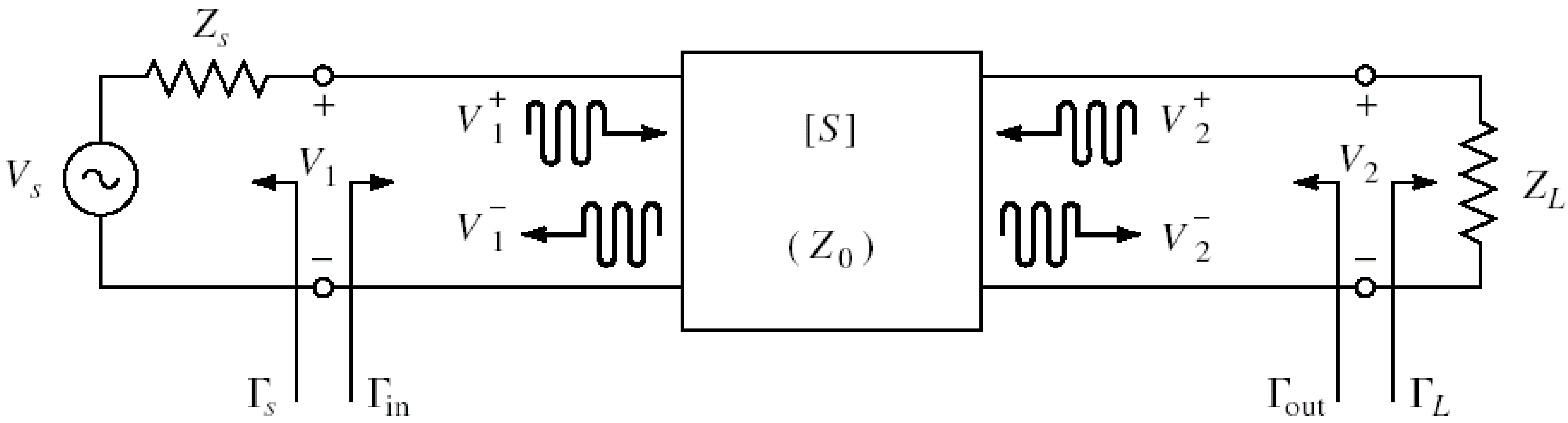
Input section constant gain circles

$$\left|\Gamma_S - \frac{g_S \cdot S_{11}^*}{1-(1-g_S)\cdot|S_{11}|^2}\right| = \frac{\sqrt{1-g_S}\cdot\left(1-|S_{11}|^2\right)}{1-(1-g_S)\cdot|S_{11}|^2} \qquad |\Gamma_S - C_S| = R_S$$

$$C_S = \frac{g_S \cdot S_{11}^*}{1-(1-g_S)\cdot|S_{11}|^2} \qquad R_S = \frac{\sqrt{1-g_S}\cdot\left(1-|S_{11}|^2\right)}{1-(1-g_S)\cdot|S_{11}|^2}$$

- Equation of a circle in the complex plane where Γ_S is plotted
- **Interpretation**: Any reflection coefficient Γ_S which plotted in the complex plane lies **on** the circle drawn for $g_{circle} = G_{circle}/G_{Smax}$ will lead to a gain $G_S = G_{circle}$
  - Any reflection coefficient Γ_S plotted **outside** this circle will lead to a gain $G_S < G_{circle}$
  - Any reflection coefficient Γ_S plotted **inside** this circle will lead to a gain $G_S > G_{circle}$

Amplifier as two-port



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  - stability
  - power gain
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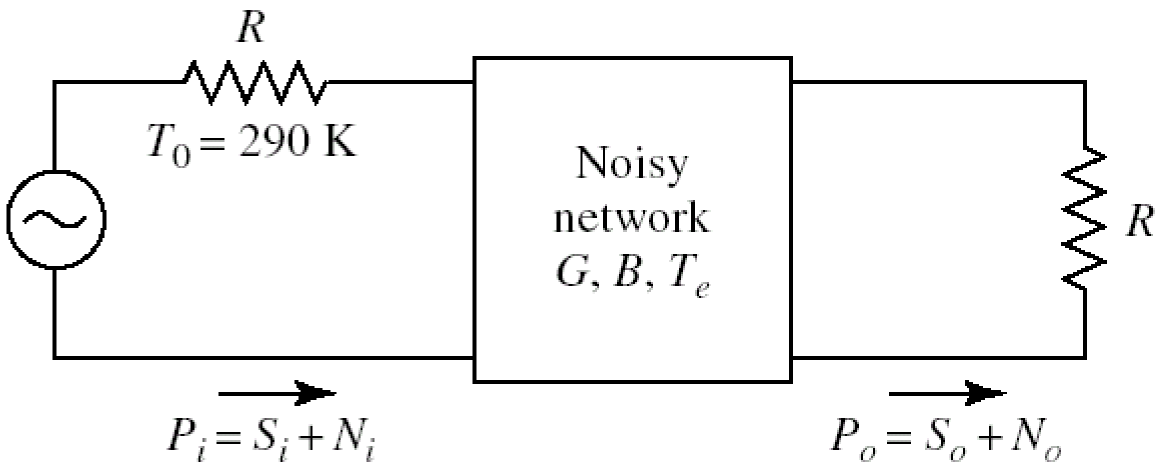
  - linearity (sometimes – large signals)

Noise Figure F



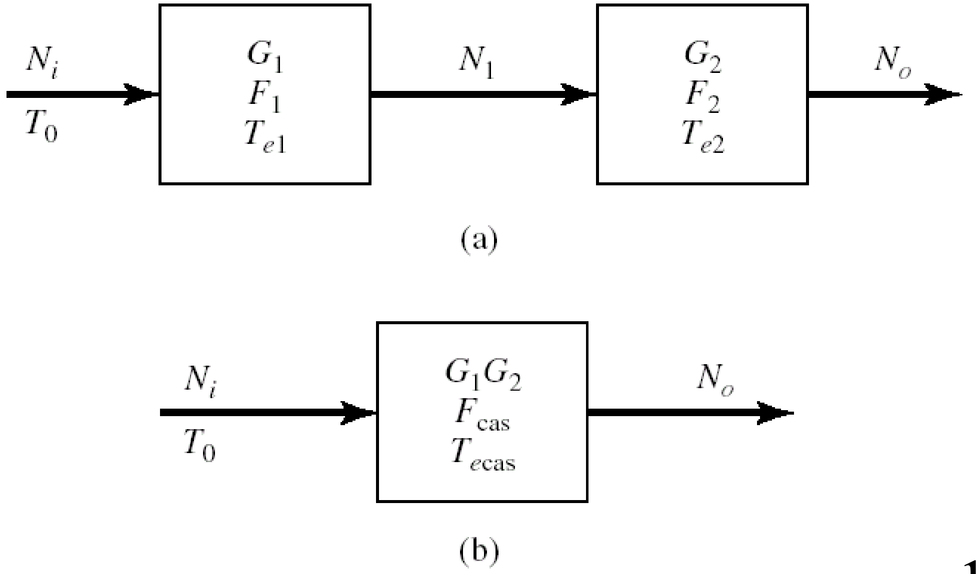
- The noise figure F, is a measure of the reduction in signal-to-noise ratio between the input and output of a device, when (by definition) the input noise power is assumed to be the noise power resulting from a matched resistor at To = 290 K (reference noise conditions)

$$F = \left.\frac{S_i / N_i}{S_o / N_o}\right|_{T_0 = 290K}$$

$$V_{n(ef)} = \sqrt{4kTBR}$$

$$P_n = kTB$$

Noise figure of a cascaded system

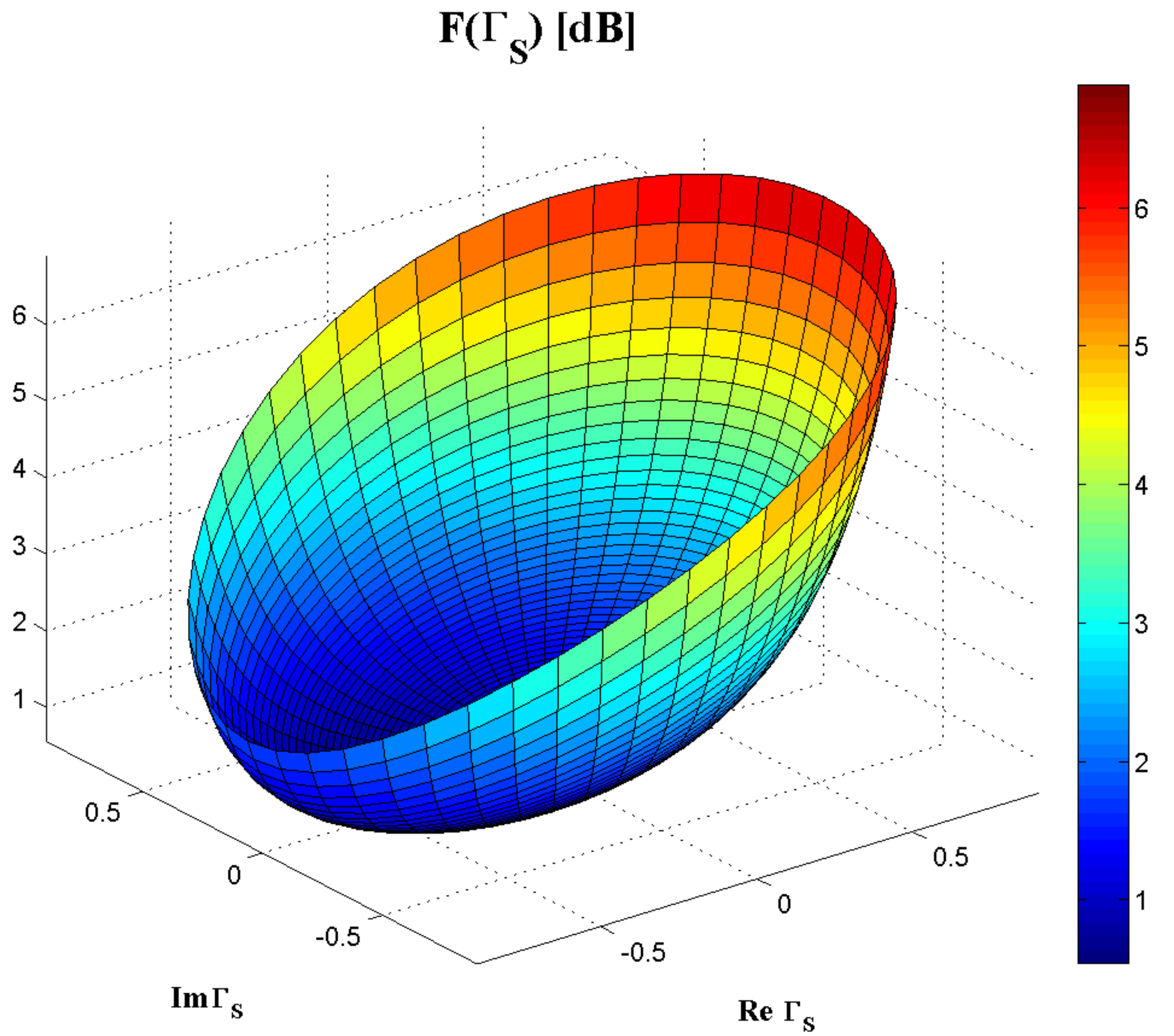


$$G_{cas} = G_1 \cdot G_2 \qquad F_{cas} = F_1 + \frac{1}{G_1}\left(F_2 - 1\right)$$

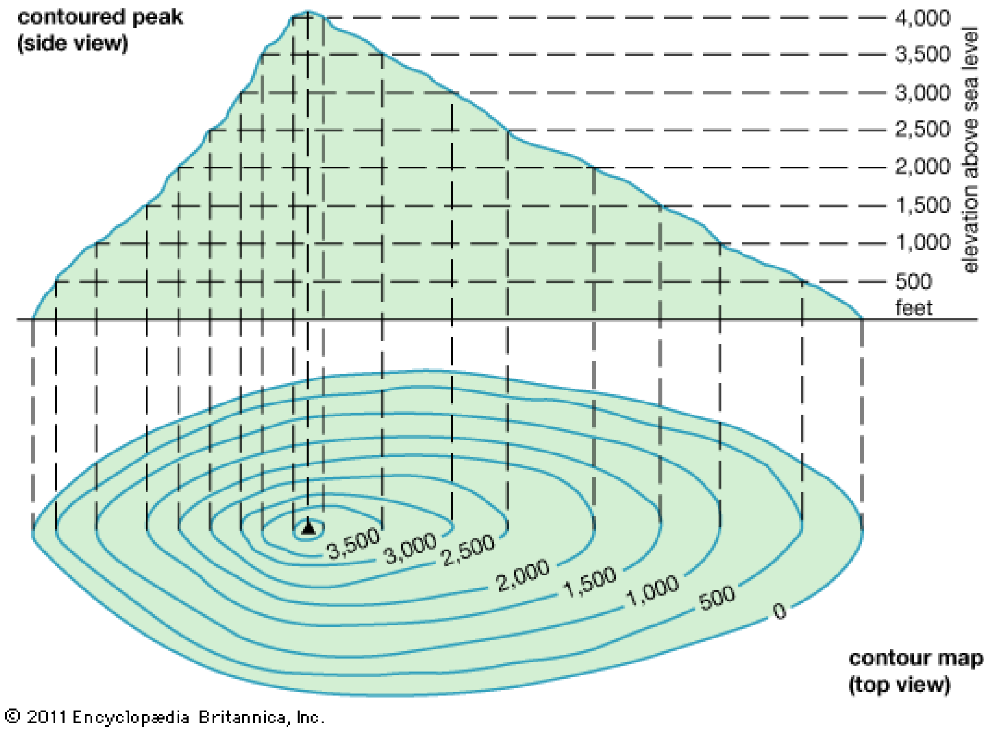
- Friis Formula (**!linear scale**)

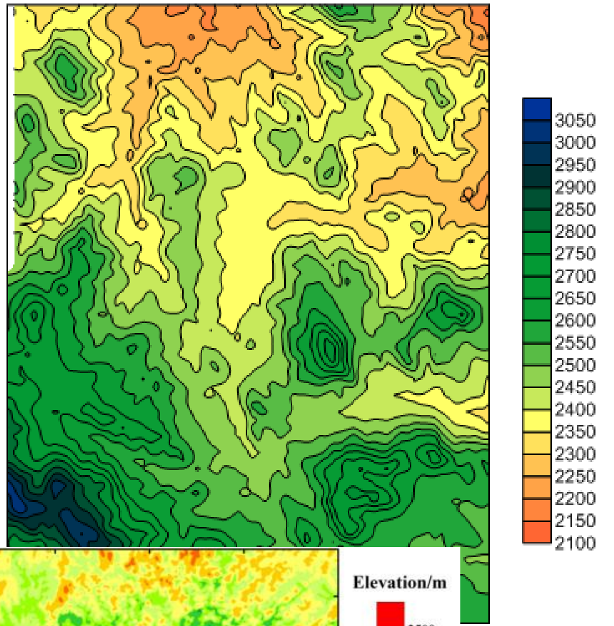
$$F_{cas} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 \cdot G_2} + \frac{F_4 - 1}{G_1 \cdot G_2 \cdot G_3} + \cdots$$

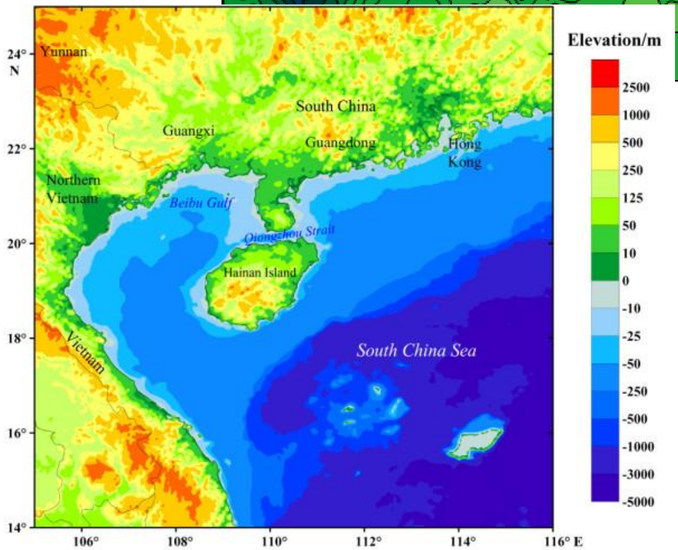
F[dB](Γ_S)



Contour map/lines







Circles of constant noise figure

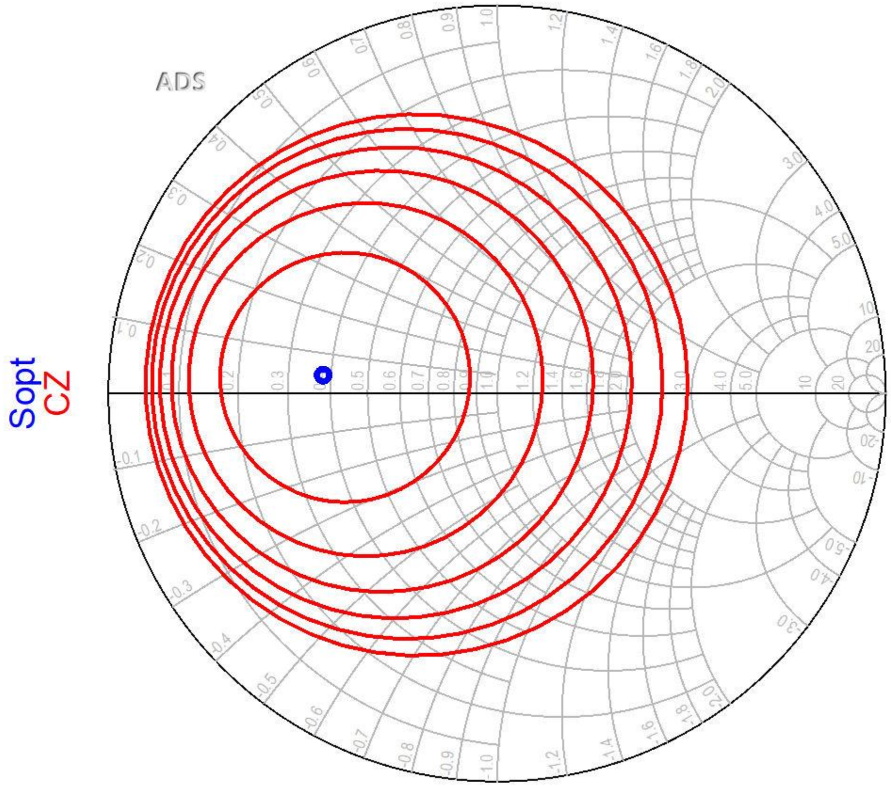
$$N = \frac{F - F_{\min}}{4 \cdot r_n} \cdot \left|1 + \Gamma_{opt}\right|^2 \qquad \left|\Gamma_S - \frac{\Gamma_{opt}}{N+1}\right| = \frac{\sqrt{N \cdot \left(N + 1 - \left|\Gamma_{opt}\right|^2\right)}}{N+1}$$

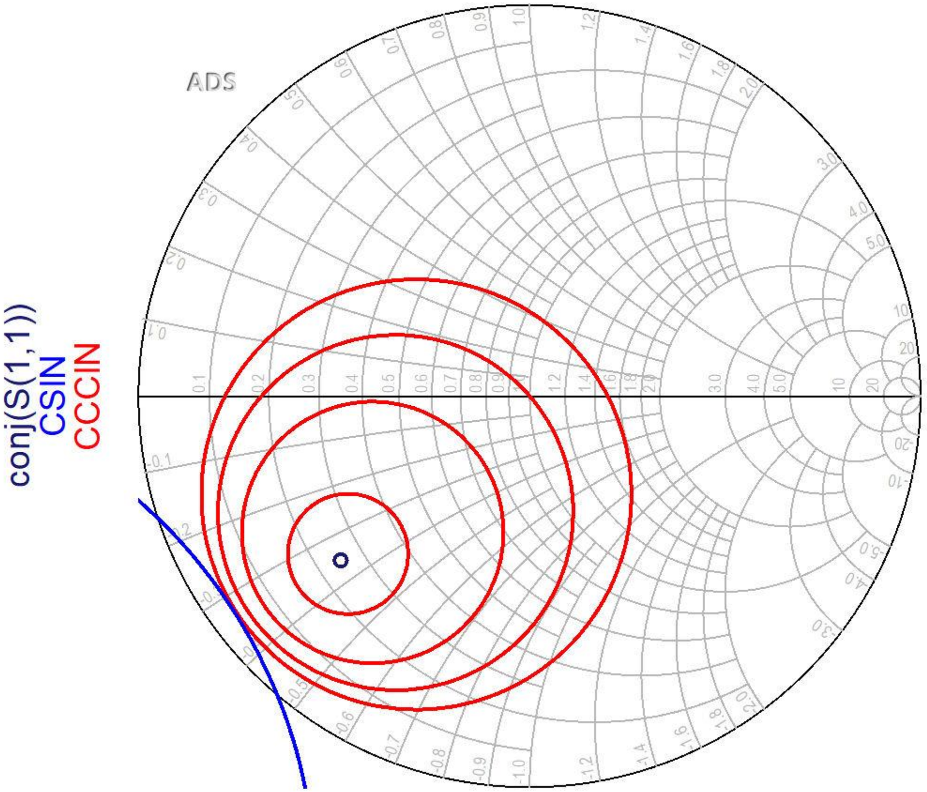
$$\left|\Gamma_S - C_F\right| = R_F \qquad C_F = \frac{\Gamma_{opt}}{N+1} \quad R_F = \frac{\sqrt{N \cdot \left(N + 1 - \left|\Gamma_{opt}\right|^2\right)}}{N+1}$$

- The locus in the complex plane Γ_S of the points with constant noise figure is a circle
- **Interpretation**: Any reflection coefficient Γ_S which plotted in the complex plane lies **on** the circle drawn for F_{circle} will lead to a noise factor $F = F_{circle}$
  - Any reflection coefficient Γ_S plotted **outside** this circle will lead to a noise factor $F > F_{circle}$
  - Any reflection coefficient Γ_S plotted **inside** this circle will lead to a noise factor $F < F_{circle}$

LNA – Low Noise Amplifier

- Usually a transistor suitable for implementing an LNA at a certain frequency will have input gain circles and noise circles in the same area for Γ_S

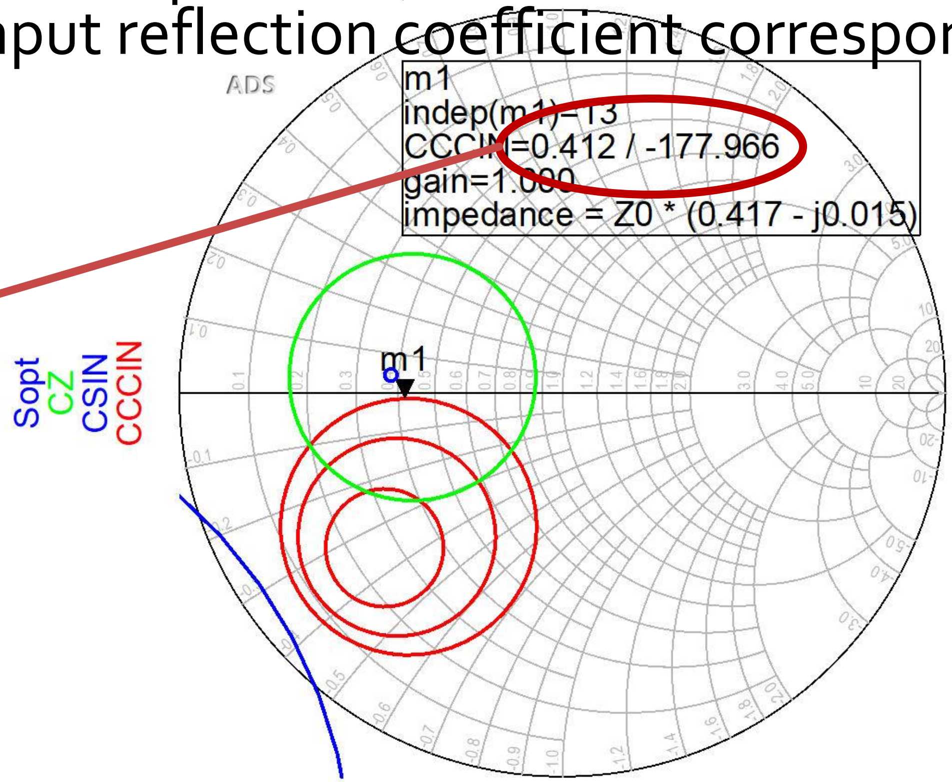




Matching – 2

- We plot on the complex plane (Smith Chart) the stability/gain/noise circles (depending on the particular application)
- We choose a point with a suitable position relative to these circles (also application dependent)
- We determine the input reflection coefficient corresponding to this point, Γ_S

$$\Gamma_S = 0.412\angle -177.966^{\circ}$$

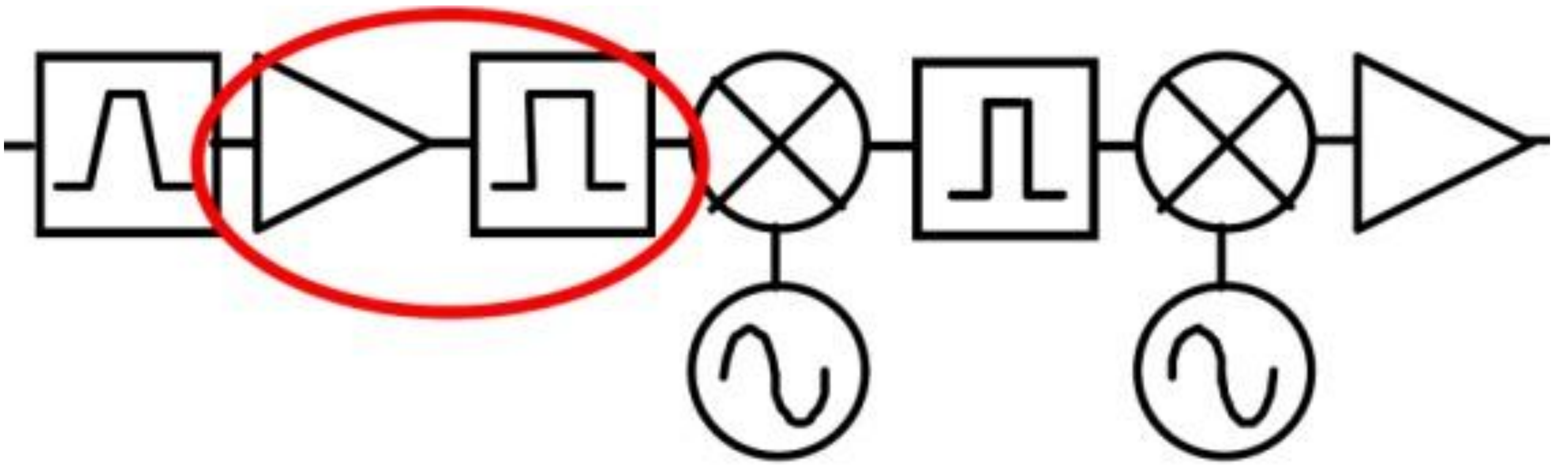


Continue

Microwave Filters

Assignment

- this structure is frequently encountered in radiocommunication systems



$$P_n = kTB$$

Microwave Filters

- Two ways of implementing filters in microwave frequency range
  - microwave specific structures (coupled lines, dielectric resonators, periodic structures)
  - **filter synthesis** with lumped elements followed by implementation with transmission lines
- the first strategy leads to more efficient filters but:
  - has lower generality
  - design is often difficult (lack of analytical relationships)

Filter synthesis

- Filter is designed with lumped elements (L/C) followed by implementation with distributed elements (transmission lines)
  - general
  - analytical relationships easy to implement on the computer
  - efficient
- The preferred procedure is **insertion loss method**

Insertion loss method

- In the insertion loss method, we are passing an intermediate stage (low pass prototype) in order to obtain any type of filter, with controlled specifications

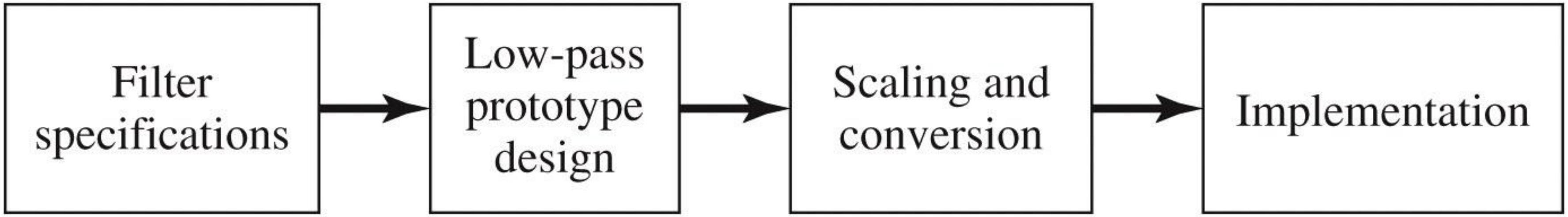


Figure 8.23

Insertion loss method

- We control the power loss ratio/attenuation introduced by the filter:
  - in the passband (pass all frequencies)
  - in the stopband (reject all frequencies)

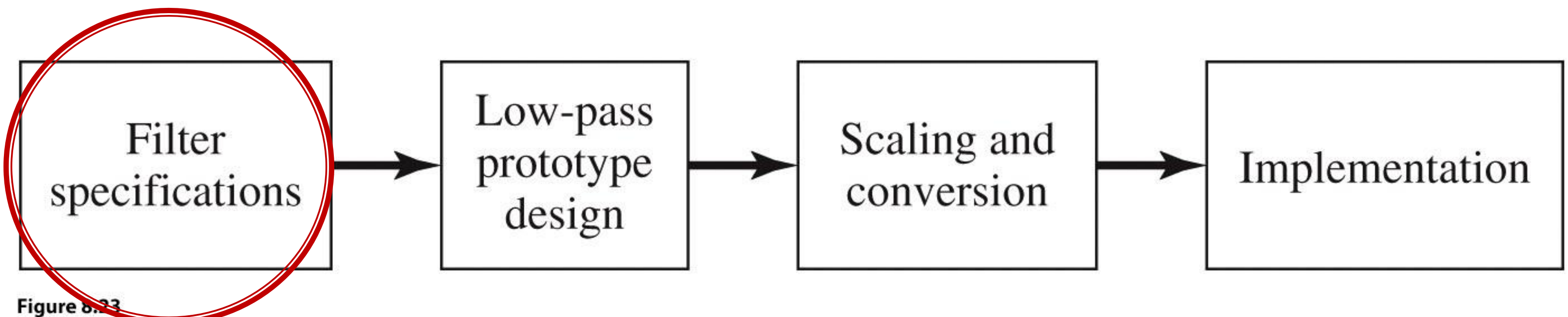


Figure 8.23

Insertion loss method

$$P_{LR} = \frac{P_S}{P_L} = \frac{1}{1 - |\Gamma(\omega)|^2}$$

- $|\Gamma(\omega)|^2$ is an even function of ω

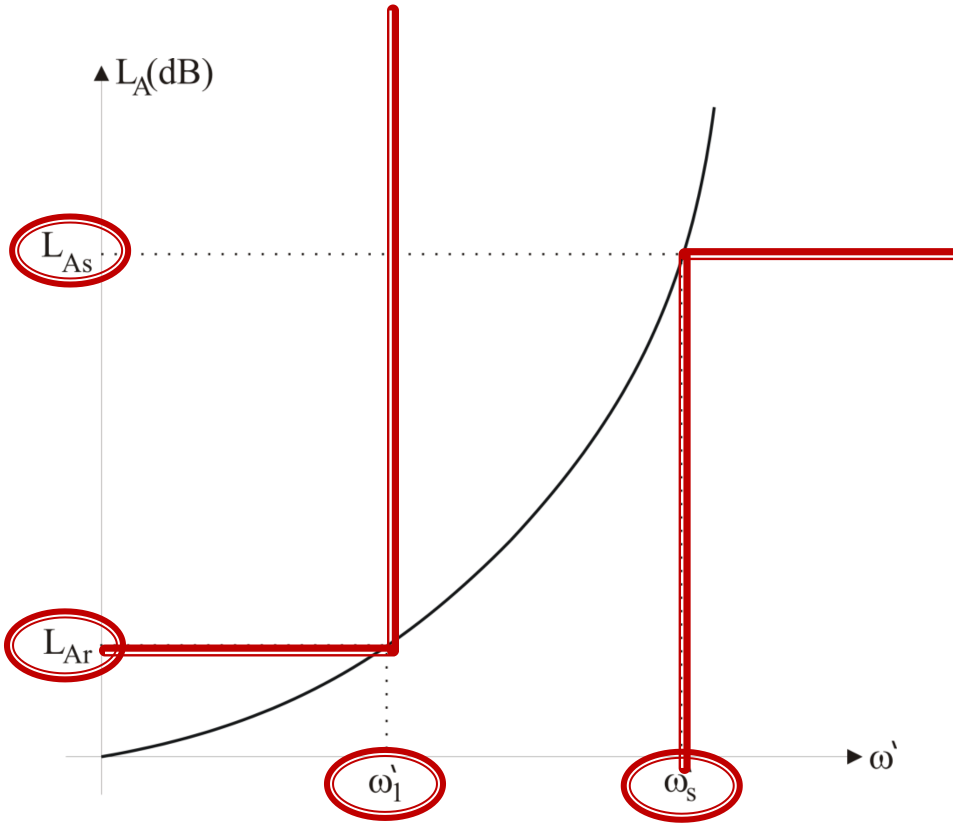
$$|\Gamma(\omega)|^2 = \frac{M(\omega^2)}{M(\omega^2) + N(\omega^2)}$$

$$P_{LR} = 1 + \frac{M(\omega^2)}{N(\omega^2)}$$

- Choosing M and N polynomials appropriately leads to a filter with a completely specified frequency response

Filter specifications

- Attenuation
  - in passband
  - in stopband
  - most often in **dB**
- Frequency range
  - passband
  - stopband
  - cutoff frequency ω_1' usually normalized (**= 1**)



Insertion loss method

- We choose the right polynomials to design an **low-pass** filter (prototype)
- The low-pass prototype are then converted to the desired other types of filters
  - low-pass, high-pass, bandpass, or bandstop

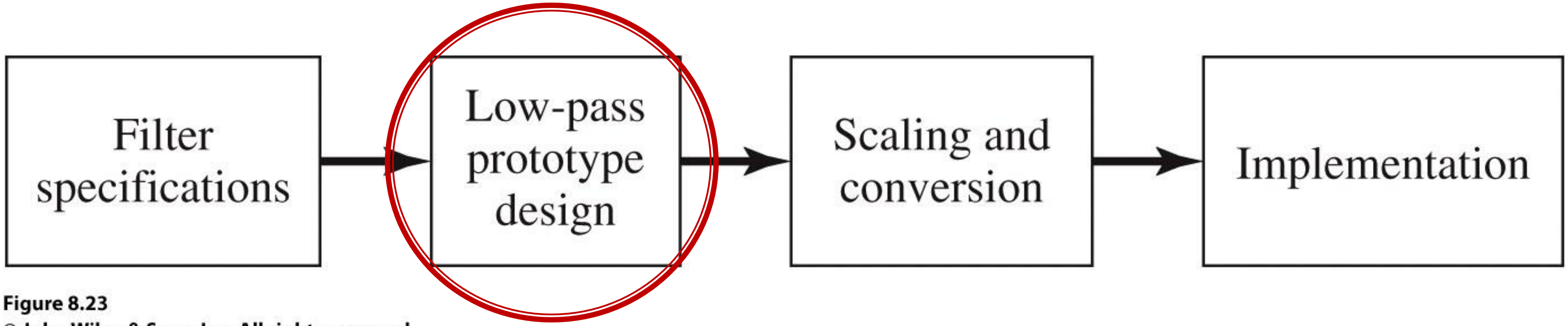


Figure 8.23

Practical low-pass prototypes responses

- **Maximally flat filters** (Butterworth, binomial): provide the flattest possible passband response
- **Equal ripple filters** (Chebyshev): provide a sharper cutoff but the passband response will have ripples
- **Elliptic function filters**, they have equal-ripple responses in the passband as well as in the stopband,
- **Linear phase filters**, offer linear phase response in the passband to avoid signal distortion (important in some applications)

Maximally Flat/Equal ripple LPF Prototype

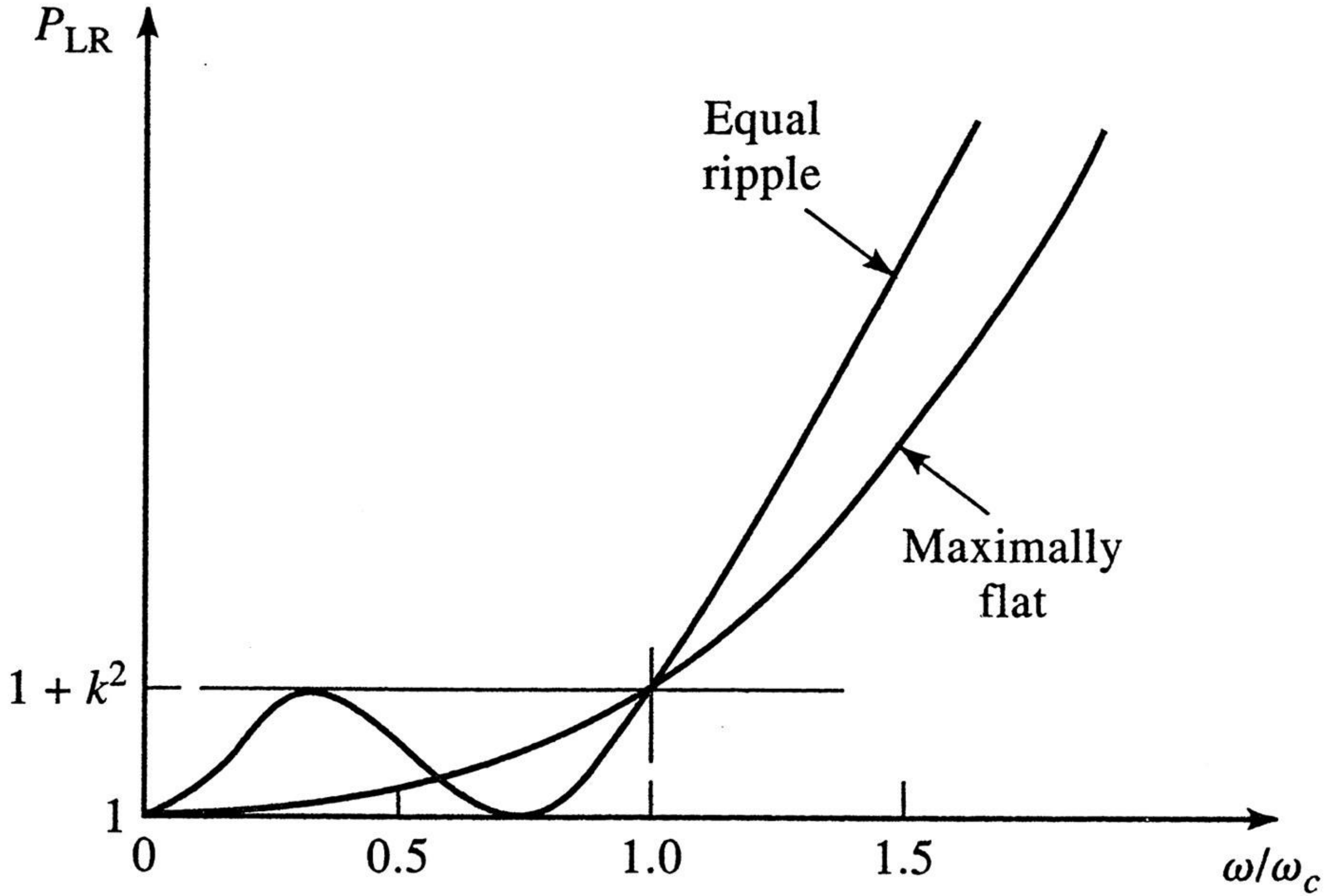


Figure 8.21

Elliptic function LPF Prototype

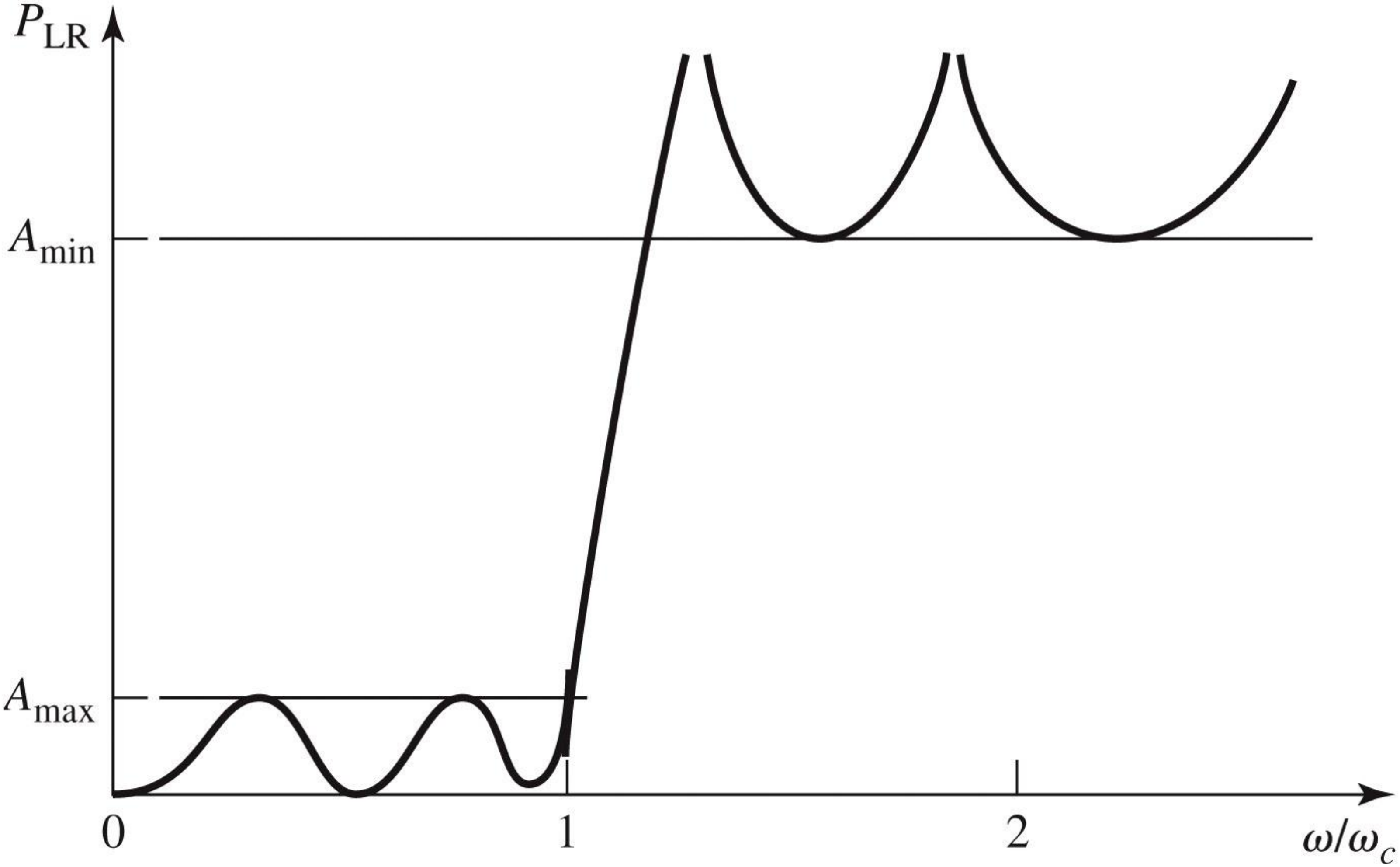


Figure 8.22

Maximally Flat LPF Prototype

- Polynomial

$$P_{LR} = 1 + k^2 \cdot \left(\frac{\omega}{\omega_c} \right)^{2N}$$

- For $\omega >> \omega_c$

$$P_{LR} \approx k^2 \cdot (\omega / \omega_c)^{2N}$$

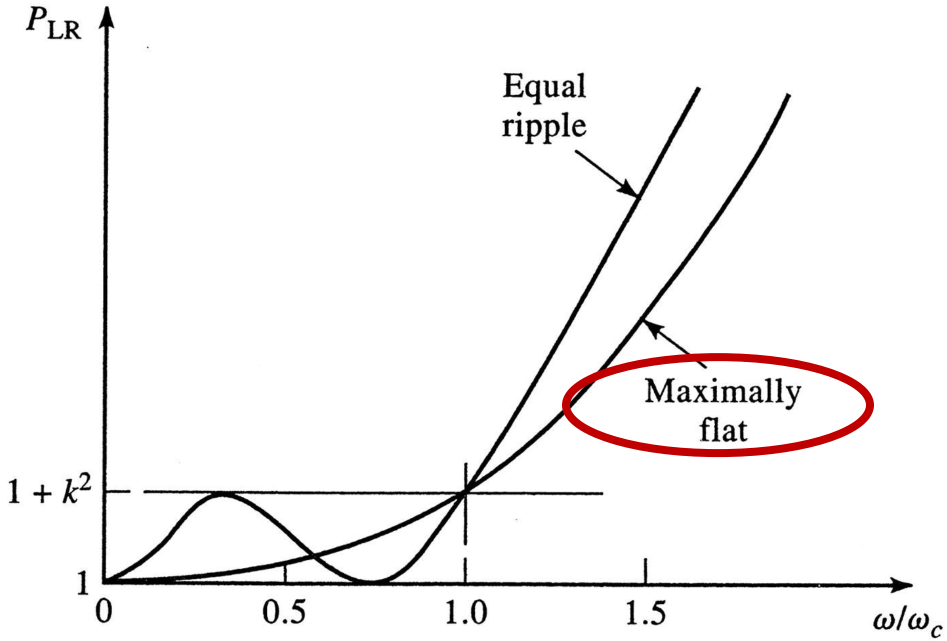


Figure 8.21

- attenuation increases at a rate of 20·N dB/decade
- k gives the attenuation at cutoff frequency (3dB cutoff imposes k = 1)

Equal Ripple LPF Prototype

- Polynomial

$$P_{LR} = 1 + k^2 \cdot T_N^2\left(\frac{\omega}{\omega_c}\right)$$

- For $\omega >> \omega_c$

$$P_{LR} \approx \frac{k^2}{4} \cdot \left(\frac{2 \cdot \omega}{\omega_c}\right)^{2N}$$

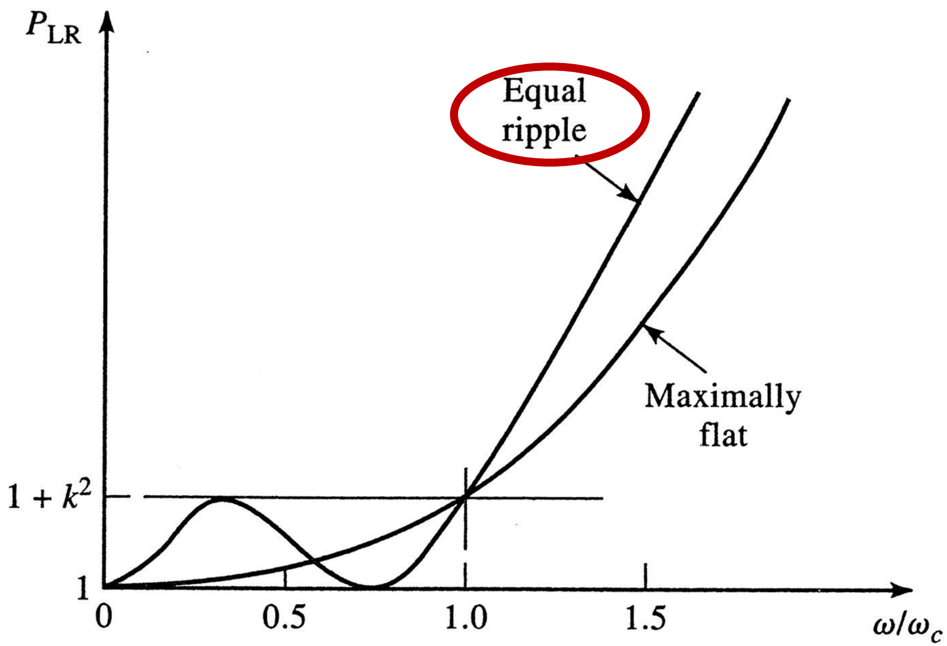
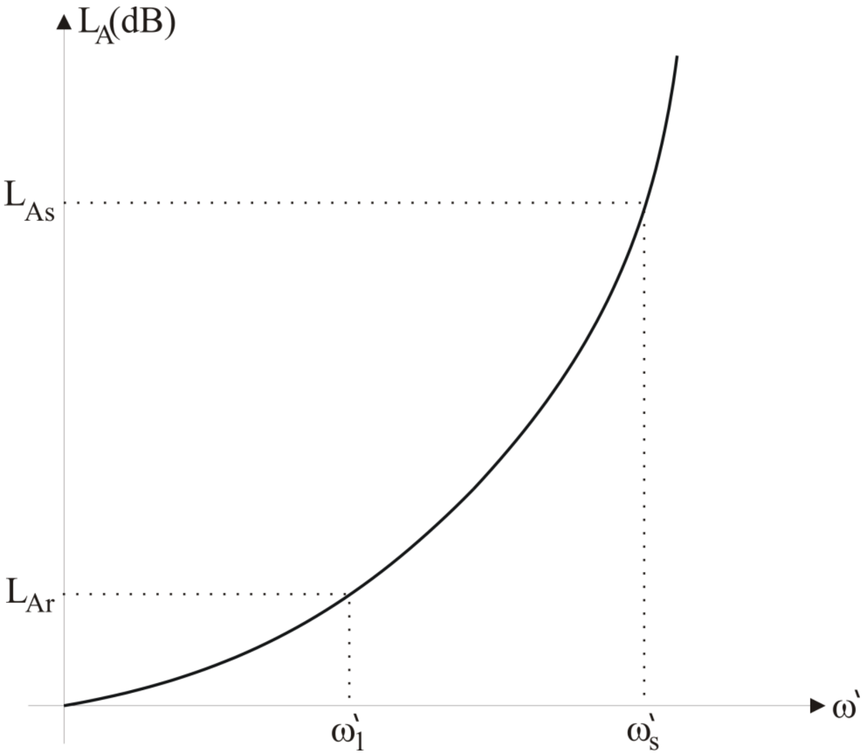


Figure 8.21

- attenuation increases at a rate of 20·N dB/decade (**also**)
- attenuation is $(2^{2N})/4$ greater than the binomial response at any given frequency where $\omega >> \omega_c$
- the passband ripples: $1 + k^2$, k gives the ripple

Order (N) of the Maximally Flat filter

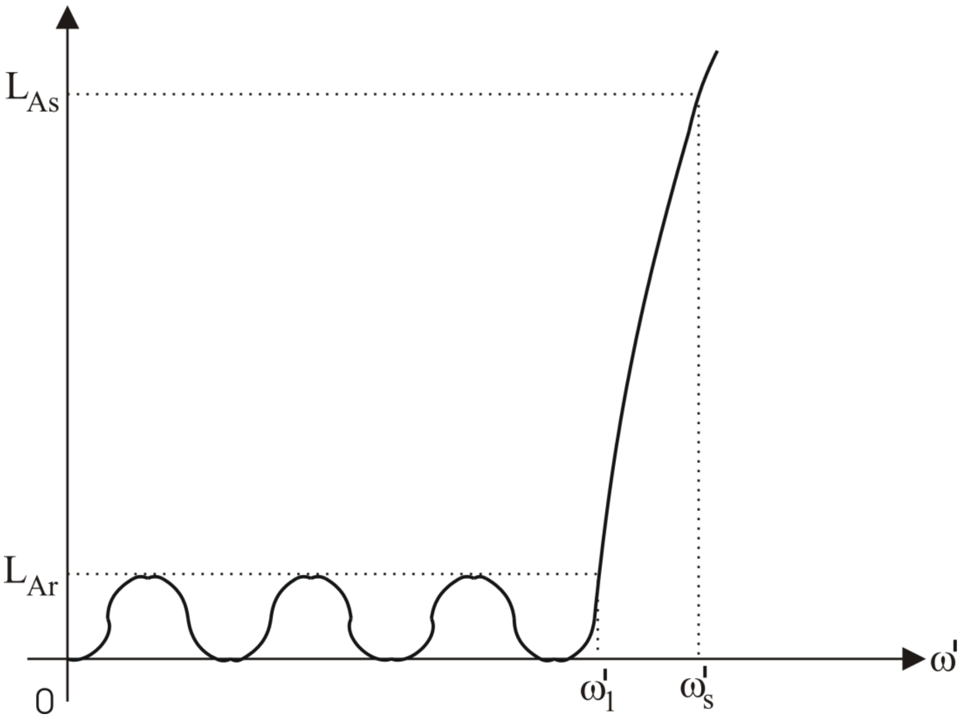
$$n \geq \frac{\log\left(\dfrac{10^{\frac{L_{As}}{10}}-1}{10^{\frac{L_{Ar}}{10}}-1}\right)}{2\cdot\log\dfrac{\omega'_s}{\omega'_1}}$$



- !attenuations in **dB**

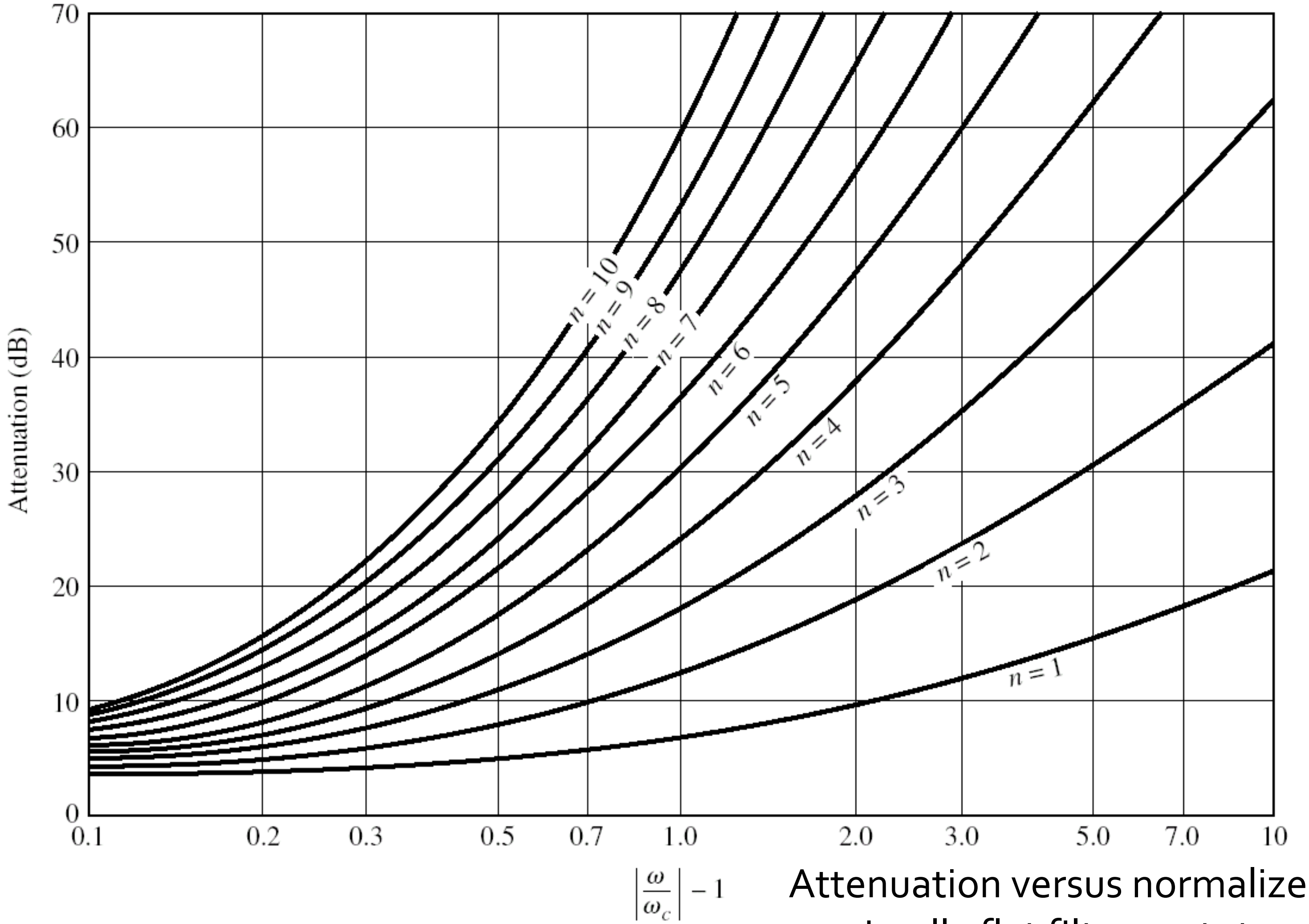
Order (N) of the Equal Ripple filter

$$n \geq \frac{\cosh^{-1}\left(\sqrt{\dfrac{10^{\frac{L_{As}}{10}}-1}{10^{\frac{L_{Ar}}{10}}-1}}\right)}{\cosh^{-1}\left(\dfrac{\omega'_s}{\omega'_1}\right)}$$



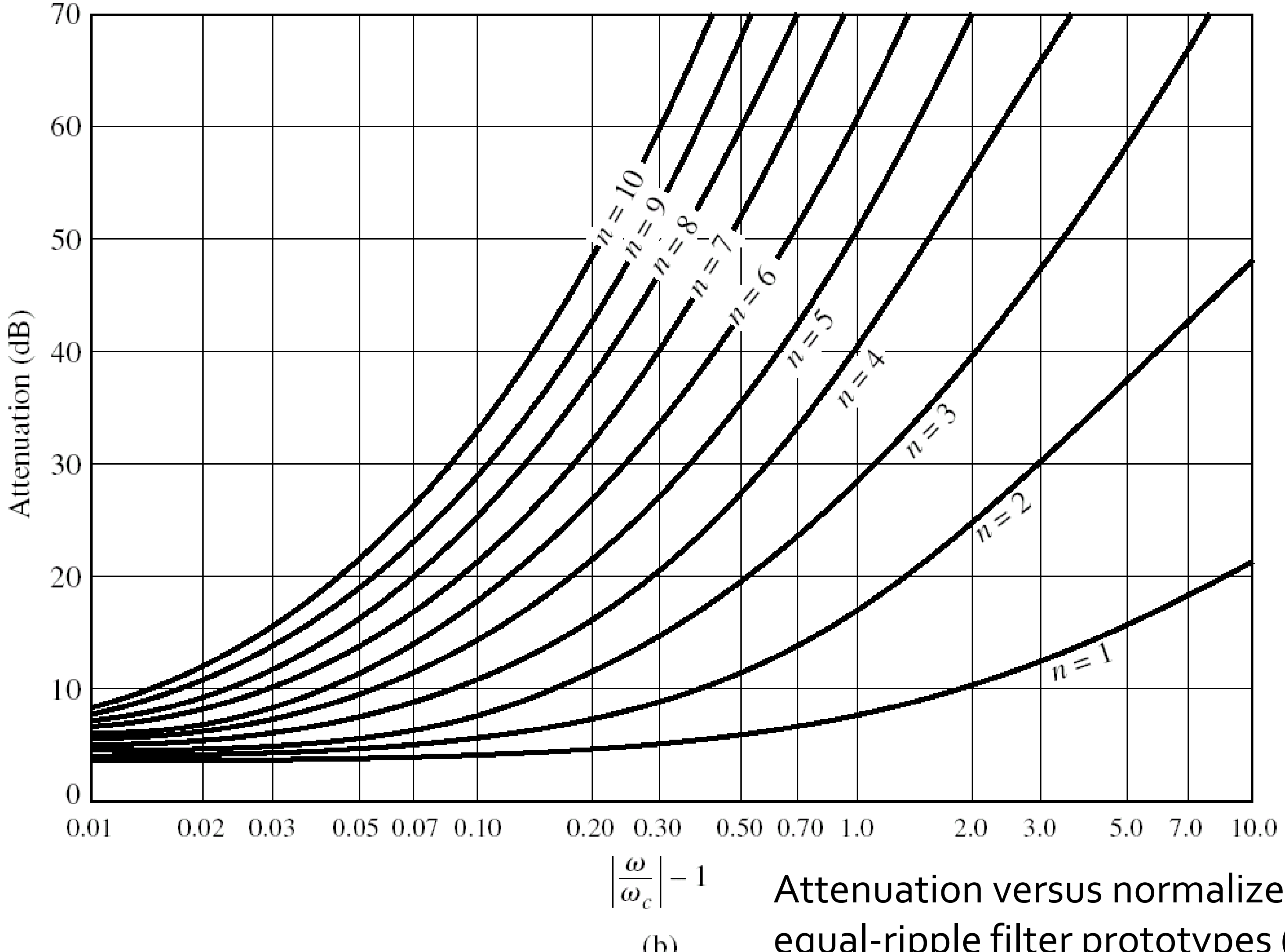
- !attenuations in **dB**

Maximally flat filter prototypes



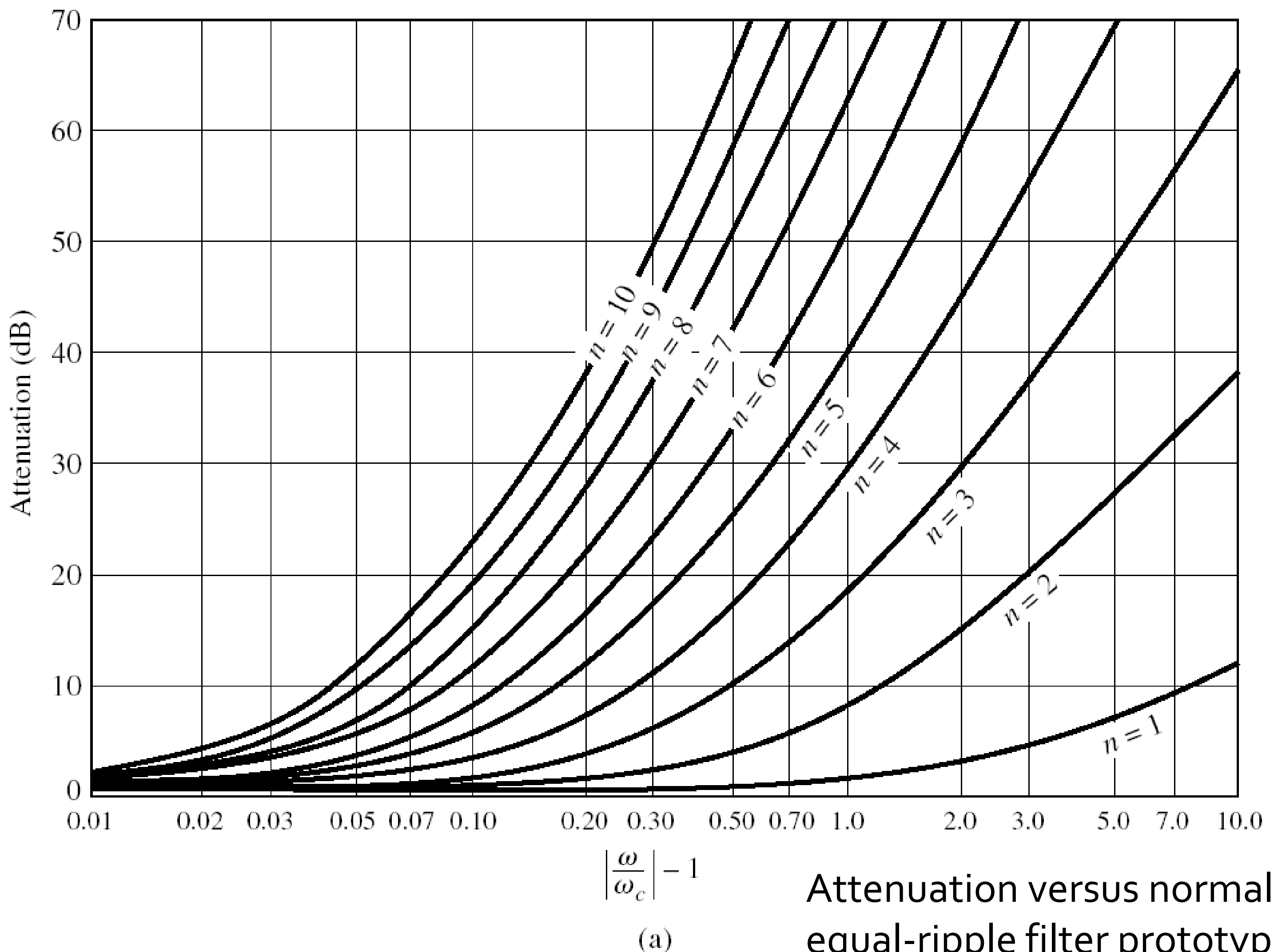
Attenuation versus normalized frequency for maximally flat filter prototypes

3 dB Equal-ripple filter prototypes



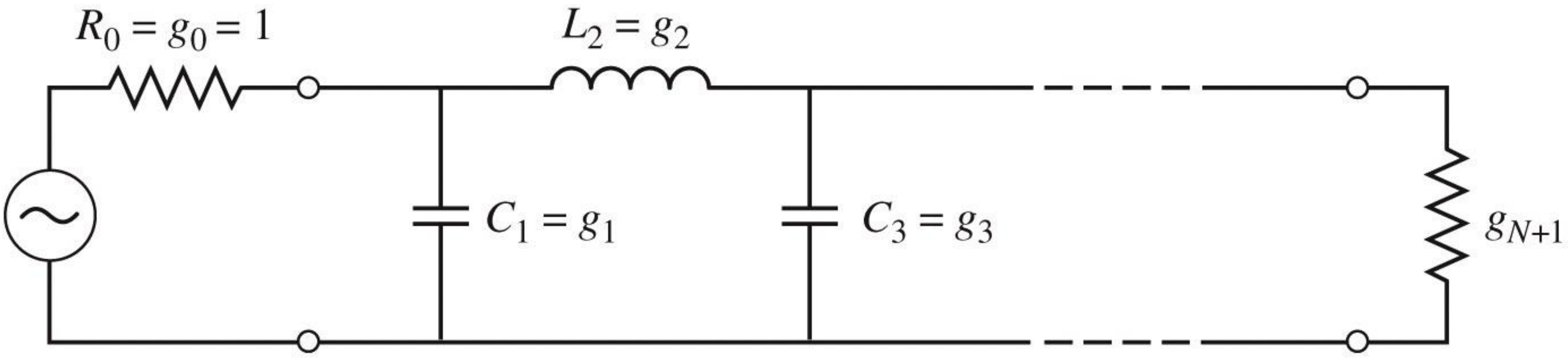
Attenuation versus normalized frequency for equal-ripple filter prototypes (3dB)

0.5 dB Equal-ripple filter prototypes

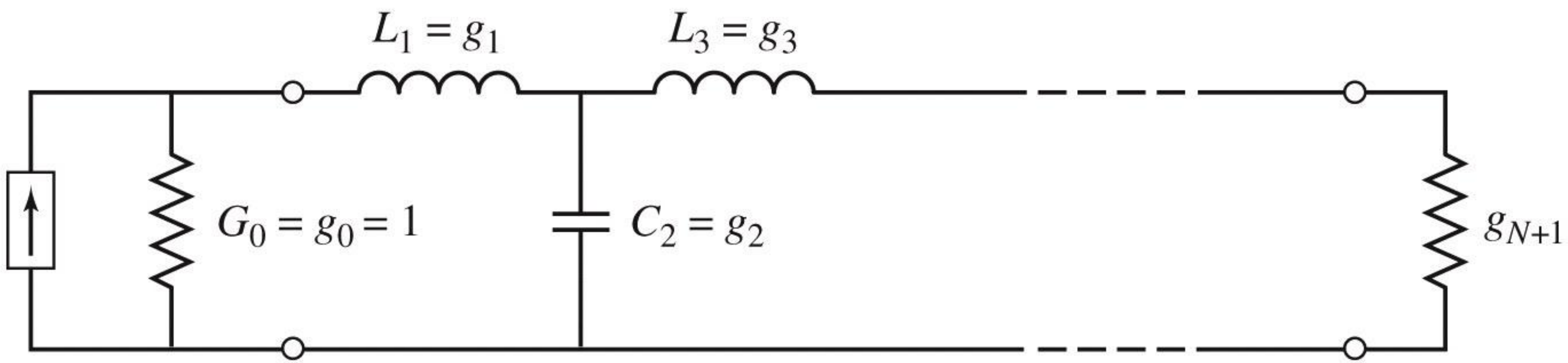


Attenuation versus normalized frequency for equal-ripple filter prototypes (0.5dB)

Prototype Filters



(a)



(b)

Prototype Filters

- Prototype filters are:
  - Low-Pass Filters (**LPF**)
  - cutoff frequency **$\omega_o = 1$ rad/s** ($f_o = 0.159$ Hz)
  - connected to a source with **$R = 1\Omega$**
- The number of reactive elements (L/C) is the order of the filter (N)
- Reactive elements are alternated: series L / shunt C
- There two prototypes with the same response, a prototype beginning with a shunt C element, and a prototype beginning with a series L element

Prototype Filters

- We define filter parameters g_i, i=0,N+1
- g_i are the element values in the prototype filter

$$g_0 = \begin{cases} \text{generator resistance } R'_0 \;\; if \;\; g_1 = C'_1 \\ \text{generator conductance } G'_0 \;\; if \;\; g_1 = L'_1 \end{cases}$$

$$g_k\Big|_{k=\overline{1,N}} = \begin{cases} \text{inductance for series inductors} \\ \text{capacitance for shunt capacitors} \end{cases}$$

$$g_{N+1} = \begin{cases} \text{load resistance } R'_{N+1} \;\; if \;\; g_N = C'_N \\ \text{load conductance } G'_{N+1} \;\; if \;\; g_N = L'_N \end{cases}$$

Maximally Flat LPF Prototype

- Formulas for filter parameters

$$g_0 = 1$$

$$g_k = 2 \cdot \sin\left[\frac{(2 \cdot k - 1) \cdot \pi}{2 \cdot N}\right] \quad , \quad k = 1, N$$

$$g_{N+1} = 1$$

Maximally Flat LPF Prototype

TABLE 8.3 Element Values for Maximally Flat Low-Pass Filter Prototypes ($g_0 = 1$, $\omega_c = 1$, $N = 1$ to 10)

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	2.0000	1.0000									
2	1.4142	1.4142	1.0000								
3	1.0000	2.0000	1.0000	1.0000							
4	0.7654	1.8478	1.8478	0.7654	1.0000						
5	0.6180	1.6180	2.0000	1.6180	0.6180	1.0000					
6	0.5176	1.4142	1.9318	1.9318	1.4142	0.5176	1.0000				
7	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450	1.0000			
8	0.3902	1.1111	1.6629	1.9615	1.9615	1.6629	1.1111	0.3902	1.0000		
9	0.3473	1.0000	1.5321	1.8794	2.0000	1.8794	1.5321	1.0000	0.3473	1.0000	
10	0.3129	0.9080	1.4142	1.7820	1.9754	1.9754	1.7820	1.4142	0.9080	0.3129	1.0000

Source: Reprinted from G. L. Matthaei, L. Young, and E. M. T. Jones, *Microwave Filters, Impedance-Matching Networks, and Coupling Structures*, Artech House, Dedham, Mass., 1980, with permission.

Table 8.3

Equal-ripple LPF Prototype

- Formulas for filter parameters (iterative)

$$a_k = \sin\left[\frac{(2\cdot k-1)\cdot\pi}{2\cdot N}\right] \quad , \quad k = 1, N$$

$$\beta = \ln\left(\coth\frac{L_{Ar}}{17.37}\right)$$

$$\gamma = \sinh\left(\frac{\beta}{2\cdot N}\right)$$

$$b_k = \gamma^2 + \sin^2\left(\frac{k\cdot\pi}{N}\right) \quad , \quad k = 1, N$$

$$g_1 = \frac{2\cdot a_1}{\gamma}$$

$$g_k = \frac{4\cdot a_{k-1}\cdot a_k}{b_{k-1}\cdot g_{k-1}} \quad , \quad k = 2, N$$

$$g_{N+1} = \begin{cases} 1 & \textit{for odd } N \\ \coth^2\left(\dfrac{\beta}{4}\right) & \textit{for even } N \end{cases}$$

TABLE 8.4 **Element Values for Equal-Ripple Low-Pass Filter Prototypes ($g_0 = 1$, $\omega_c = 1$, $N = 1$ to 10, 0.5 dB and 3.0 dB ripple)**

0.5 dB Ripple

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	0.6986	1.0000									
2	1.4029	0.7071	1.9841								
3	1.5963	1.0967	1.5963	1.0000							
4	1.6703	1.1926	2.3661	0.8419	1.9841						
5	1.7058	1.2296	2.5408	1.2296	1.7058	1.0000					
6	1.7254	1.2479	2.6064	1.3137	2.4758	0.8696	1.9841				
7	1.7372	1.2583	2.6381	1.3444	2.6381	1.2583	1.7372	1.0000			
8	1.7451	1.2647	2.6564	1.3590	2.6964	1.3389	2.5093	0.8796	1.9841		
9	1.7504	1.2690	2.6678	1.3673	2.7239	1.3673	2.6678	1.2690	1.7504	1.0000	
10	1.7543	1.2721	2.6754	1.3725	2.7392	1.3806	2.7231	1.3485	2.5239	0.8842	1.9841

3.0 dB Ripple

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	1.9953	1.0000									
2	3.1013	0.5339	5.8095								
3	3.3487	0.7117	3.3487	1.0000							
4	3.4389	0.7483	4.3471	0.5920	5.8095						
5	3.4817	0.7618	4.5381	0.7618	3.4817	1.0000					
6	3.5045	0.7685	4.6061	0.7929	4.4641	0.6033	5.8095				
7	3.5182	0.7723	4.6386	0.8039	4.6386	0.7723	3.5182	1.0000			
8	3.5277	0.7745	4.6575	0.8089	4.6990	0.8018	4.4990	0.6073	5.8095		
9	3.5340	0.7760	4.6692	0.8118	4.7272	0.8118	4.6692	0.7760	3.5340	1.0000	
10	3.5384	0.7771	4.6768	0.8136	4.7425	0.8164	4.7260	0.8051	4.5142	0.6091	5.8095

Source: Reprinted from G. L. Matthaei, L. Young, and E. M. T. Jones, *Microwave Filters, Impedance-Matching Networks, and Coupling Structures*, Artech House, Dedham, Mass.,1980, with permission.

Table 8.4

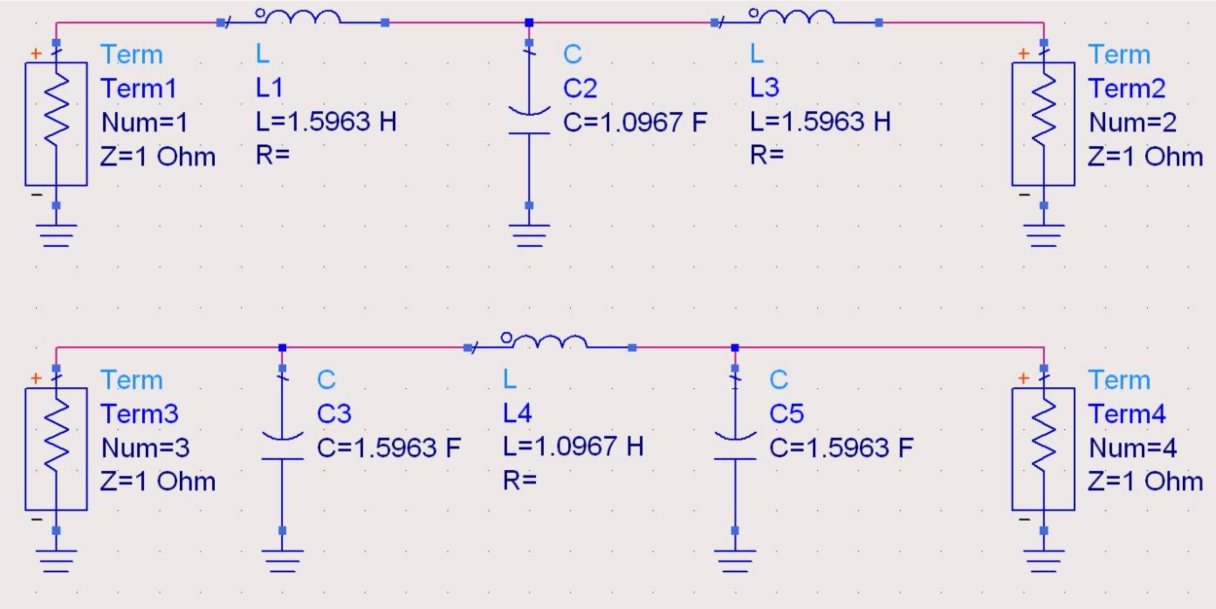
- For even N order of the filter (N = 2, 4, 6, 8 …) equal-ripple filters **must** closed by a load impedance **$g_{N+1} \neq 1$**
- If the application doesn't allow this, supplemental impedance matching is required (quarter-wave transformer, binomial …) to **$g_L = 1$**

Example

- Design a **3rd order** ~~bandpass~~ filter with **0.5 dB ripples** in passband. ~~The center frequency of the filter should be 1 GHz. The fractional bandwidth of the passband should be 10%, and the impedance 50Ω~~.

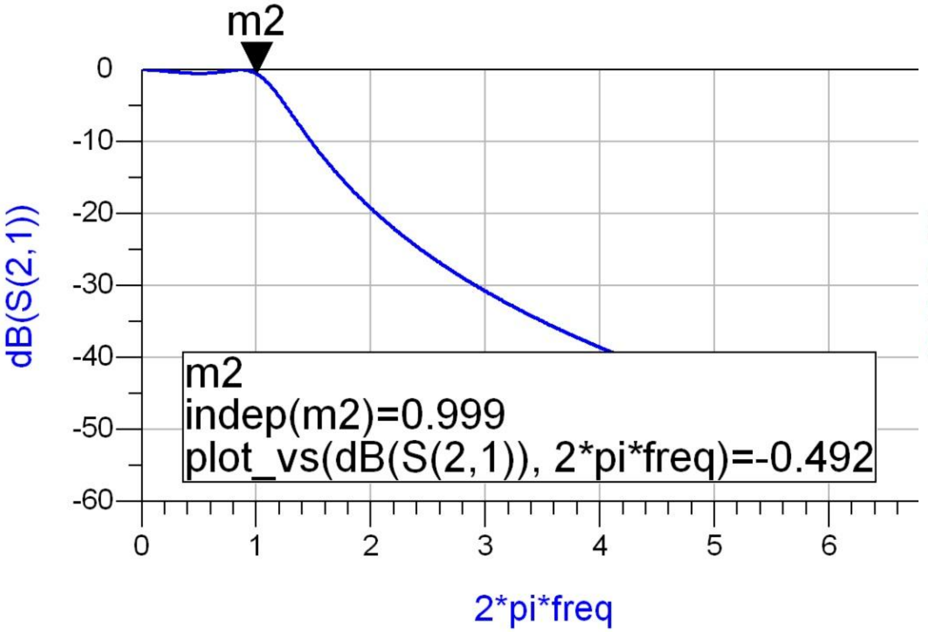
LPF Prototype

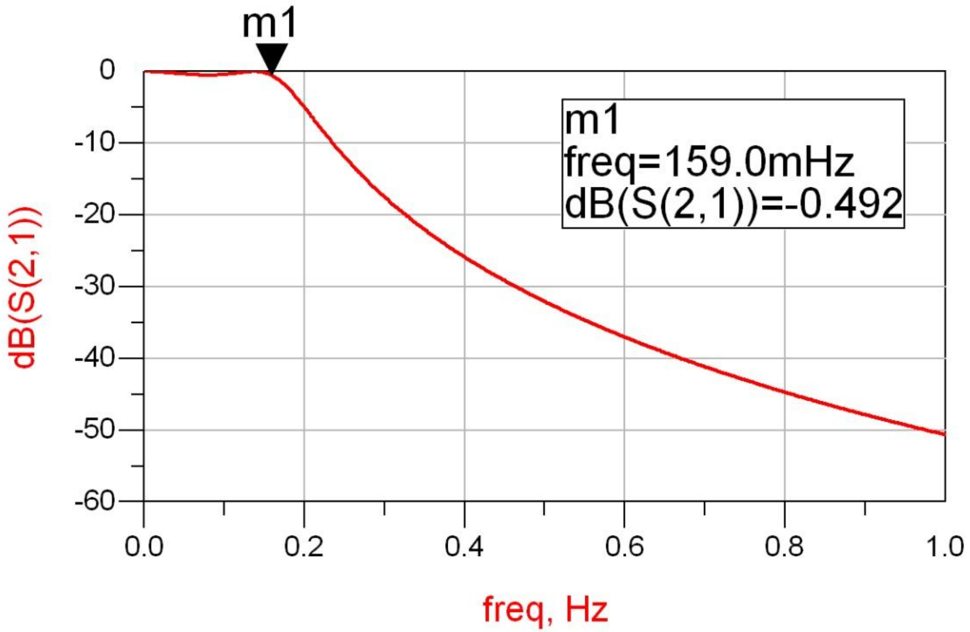
- 0.5dB equal-ripple table or design formulas:
  - g1 = 1.5963 = L1/C3,
  - g2 = 1.0967 = C2/L4,
  - g3 = 1.5963 = L3/C5,
  - g4=1.000 =R_L



LPF Prototype

- ω_o = 1 rad/s ($f_o = \omega_o / 2\pi$ = 0.159 Hz)





Impedance and Frequency Scaling

- After computing prototype filter's elements:
  - Low-Pass Filters (**LPF**)
  - cutoff frequency **ω_o = 1 rad/s** (f_o = 0.159 Hz)
  - connected to a source with **R = 1Ω**
- component values can be scaled in terms of impedance and frequency

Impedance and Frequency Scaling

- LPF Prototype is only used as an intermediate step
  - Low-Pass Filter (LPF)
  - cutoff frequency $\omega_o = 1$ rad/s ($f_o = 0.159$ Hz)
  - connected to a source with $R = 1\Omega$

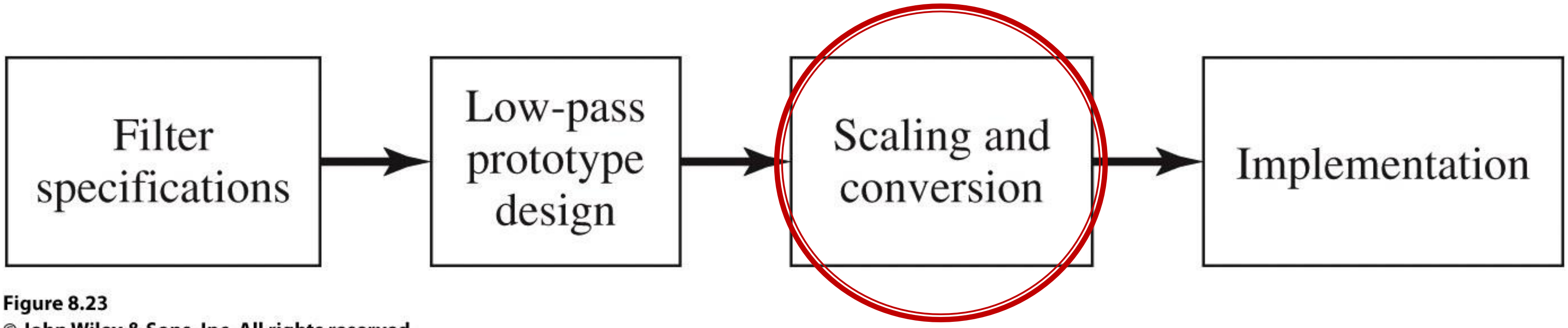


Figure 8.23

Impedance Scaling

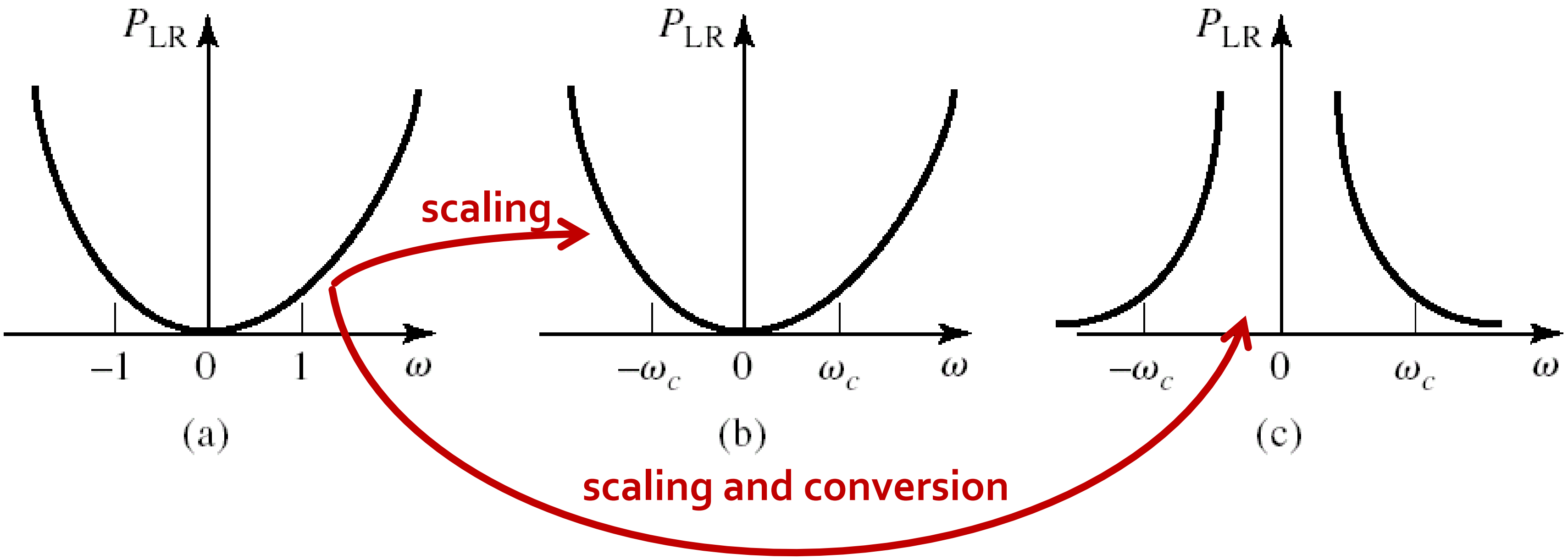
- To design a filter which will work with a source resistance of R_o we multiplying all the impedances of the prototype design by R_o ("**'**" denotes scaled values)

$$R'_s = R_0 \cdot (R_s = 1) \qquad R'_L = R_0 \cdot R_L$$

$$L' = R_0 \cdot L \qquad C' = \frac{C}{R_0}$$

Frequency Scaling

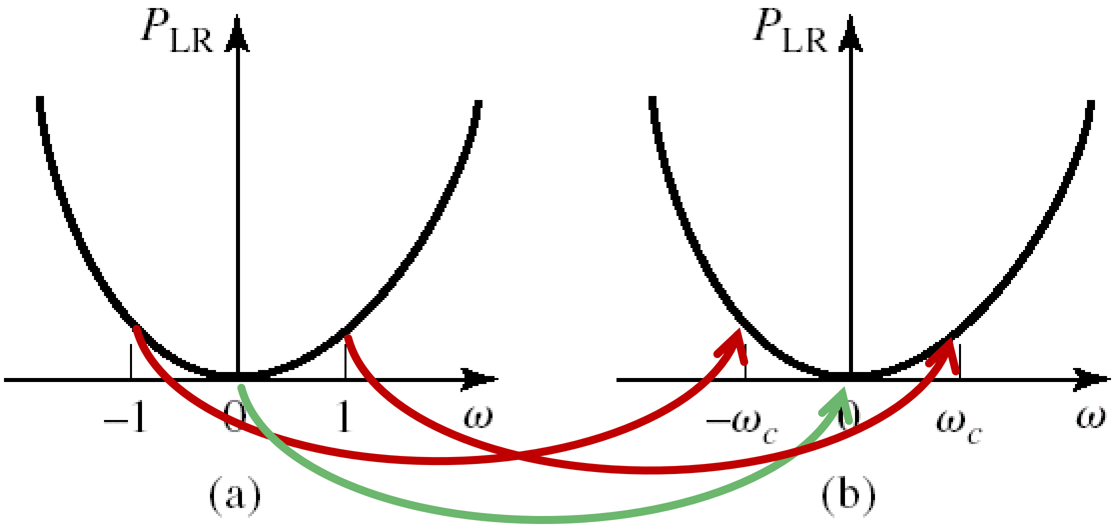
- changing the cutoff frequency – (fig. b)
- changing the type (for example LPF → HPF – fig. c) requires also conversion



Frequency Scaling

- To change the cutoff frequency of a low-pass prototype from unity to ω_c we apply a variable substitution

$$\omega \leftarrow \frac{\omega}{\omega_c}$$



Frequency Scaling

- To change the cutoff frequency of a low-pass prototype we apply a variable substitution:

$$\omega \leftarrow \frac{\omega}{\omega_c}$$

- Equivalent to the widening of the power loss filter response

$$P'_{LR}(\omega) = P_{LR}\left(\frac{\omega}{\omega_c}\right)$$

$$j \cdot X_k = j \cdot \frac{\omega}{\omega_c} \cdot L_k = j \cdot \omega \cdot L'_k \qquad j \cdot B_k = j \cdot \frac{\omega}{\omega_c} \cdot C_k = j \cdot \omega \cdot C'_k$$

Frequency Scaling LPF → LPF

- New element values for frequency scaling:

$$L'_k = \frac{L_k}{\omega_c} \qquad C'_k = \frac{C_k}{\omega_c}$$

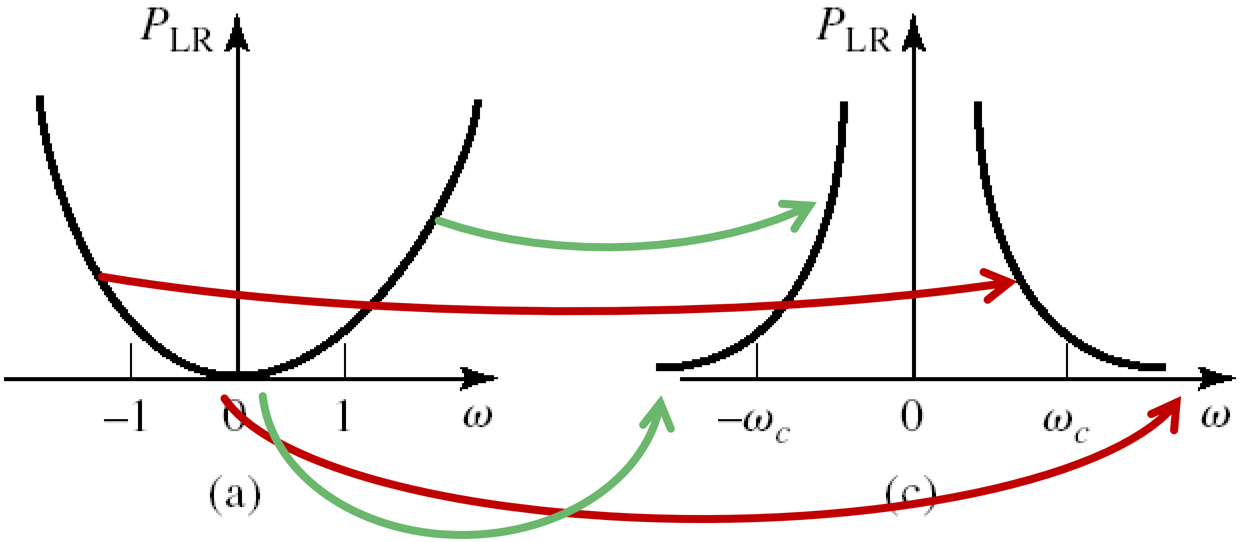
- When both impedance and frequency scaling are required:

$$L'_k = \frac{R_0 \cdot L_k}{\omega_c} \qquad C'_k = \frac{C_k}{R_0 \cdot \omega_c}$$

Low-pass to high-pass transformation LPF → HPF

- Variable substitution for LPF → HPF:

$$\omega \leftarrow -\frac{\omega_c}{\omega}$$



High-pass transformation LPF → HPF

- Variable substitution for LPF → HPF :

$$\omega \leftarrow -\frac{\omega_c}{\omega}$$

$$j \cdot X_k = -j \cdot \frac{\omega_c}{\omega} \cdot L_k = \frac{1}{j \cdot \omega \cdot C'_k} \qquad j \cdot B_k = -j \cdot \frac{\omega_c}{\omega} \cdot C_k = \frac{1}{j \cdot \omega \cdot L'_k}$$

- Impedance scaling can be included

$$C'_k = \frac{1}{R_0 \cdot \omega_c \cdot L_k} \qquad L'_k = \frac{R_0}{\omega_c \cdot C_k}$$

- In the schematic series **inductors** must be replaced with series **capacitors**, and shunt **capacitors** must be replaced with shunt **inductors**

Bandpass Transformation LPF → BPF

- Variable substitution for LPF → BPF:

$$\omega \leftarrow \frac{\omega_0}{\omega_2 - \omega_1}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) = \frac{1}{\Delta}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)$$

- where we use the fractional bandwidth of the passband and the center frequency

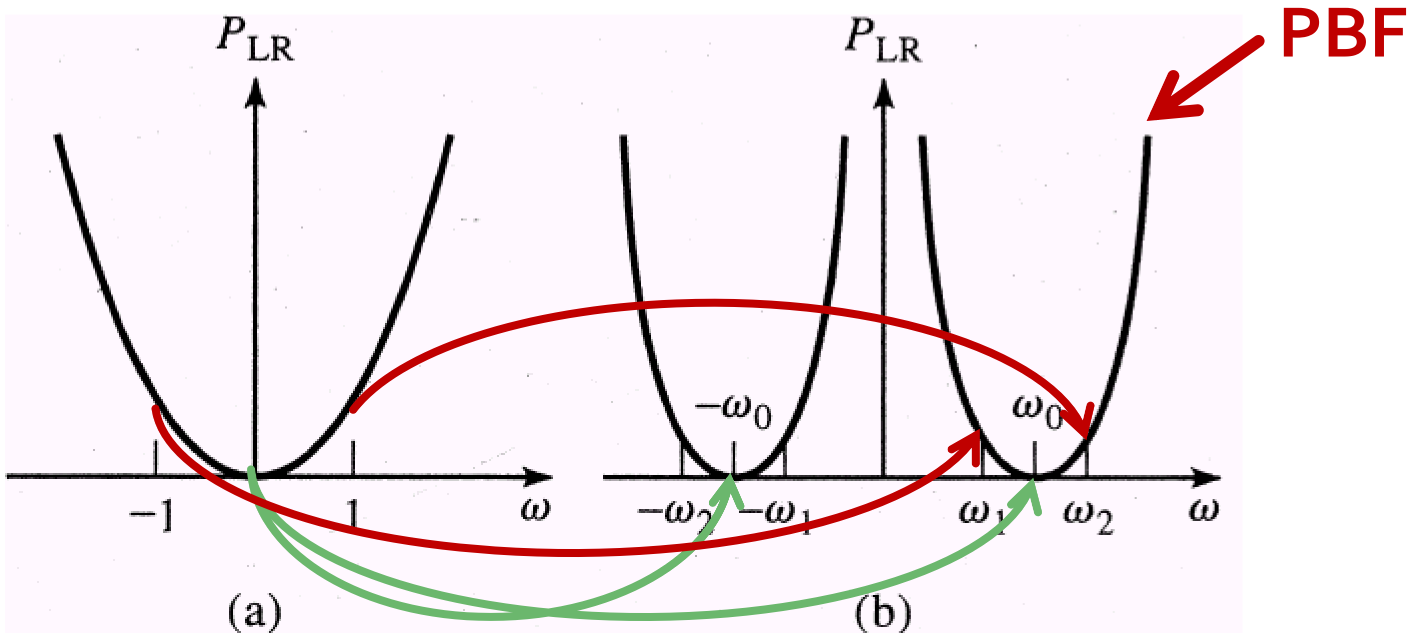
$$\Delta = \frac{\omega_2 - \omega_1}{\omega_0} \qquad \omega_0 = \sqrt{\omega_1 \cdot \omega_2}$$

Bandpass Transformation LPF → BPF

$$\omega=\omega_0 \rightarrow \frac{1}{\Delta}\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)=\frac{1}{\Delta}\left(\frac{\omega_0}{\omega_0}-\frac{\omega_0}{\omega_0}\right)=0 \qquad \omega=-\omega_0 \rightarrow \frac{1}{\Delta}\left(\frac{-\omega_0}{\omega_0}-\frac{\omega_0}{-\omega_0}\right)=0$$

$$\omega=\omega_1 \rightarrow \frac{1}{\Delta}\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)=\frac{1}{\Delta}\left(\frac{\omega_1^2-\omega_0^2}{\omega_0\cdot\omega_1}\right)=-1$$

$$\omega=\omega_2 \rightarrow \frac{1}{\Delta}\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)=\frac{1}{\Delta}\left(\frac{\omega_2^2-\omega_0^2}{\omega_0\cdot\omega_2}\right)=1$$



Bandpass Transformation LPF → BPF

$$j\cdot X_k = \frac{j}{\Delta}\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)\cdot L_k = j\cdot\frac{\omega\cdot L_k}{\Delta\cdot\omega_0} - j\cdot\frac{\omega_0\cdot L_k}{\Delta\cdot\omega} = j\cdot\omega\cdot L_k' - j\frac{1}{\omega\cdot C_k'}$$

$$j\cdot B_k = \frac{j}{\Delta}\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)\cdot C_k = j\cdot\frac{\omega\cdot C_k}{\Delta\cdot\omega_0} - j\cdot\frac{\omega_0\cdot C_k}{\Delta\cdot\omega} = j\cdot\omega\cdot C_k' - j\frac{1}{\omega\cdot L_k'}$$

- A series **inductor** in the prototype filter is transformed to a **series LC** circuit **in series**

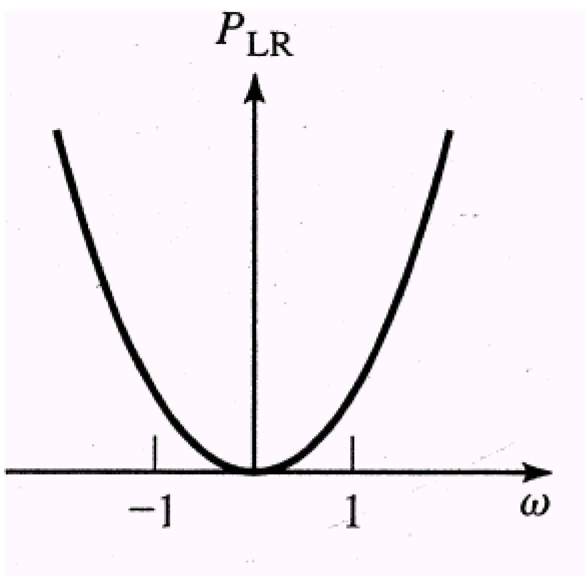
$$L_k' = \frac{L_k}{\Delta\cdot\omega_0} \qquad C_k' = \frac{\Delta}{\omega_0\cdot L_k}$$

- A shunt **capacitor** in the prototype filter is transformed to a **shunt LC** circuit **in parallel**

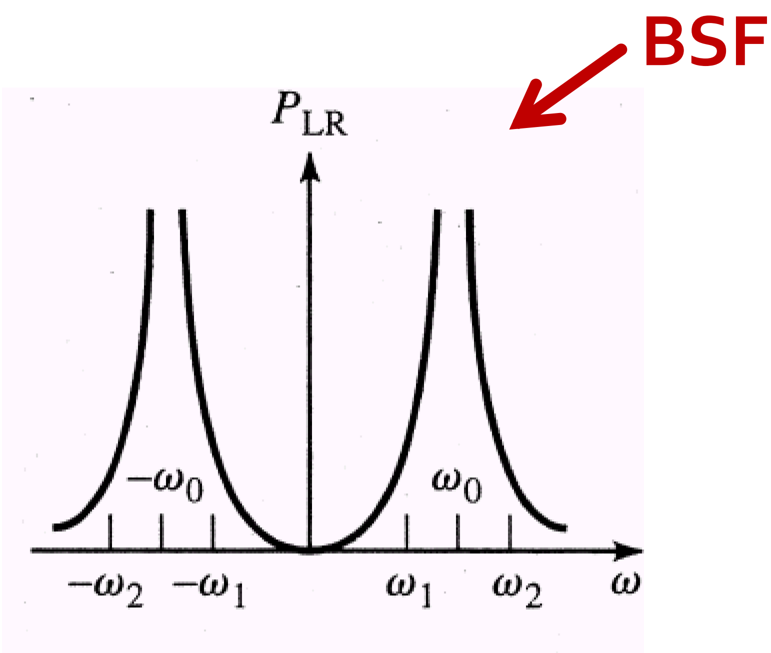
$$L_k' = \frac{\Delta}{C_k\cdot\omega_0} \qquad C_k' = \frac{C_k}{\omega_0\cdot\Delta}$$

Bandstop Transformation LPF → BSF

$$\omega \leftarrow -\Delta \cdot \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right)^{-1} \qquad \omega = \omega_0 \rightarrow \frac{-\Delta}{\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right)} = \frac{-\Delta}{\left(\frac{\omega_0}{\omega_0} - \frac{\omega_0}{\omega_0} \right)} \rightarrow \pm\infty$$



(a)



(c)

Bandstop Transformation LPF → BSF

$$\omega \leftarrow -\Delta \cdot \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right)^{-1}$$

- A series **inductor** in the prototype filter is transformed to a **shunt LC** circuit **in series**

$$L'_k = \frac{\Delta \cdot L_k}{\omega_0} \qquad C'_k = \frac{1}{\omega_0 \cdot \Delta \cdot L_k}$$

- A shunt **capacitor** in the prototype filter is transformed to a **series LC** circuit **in parallel**

$$L'_k = \frac{1}{\Delta \cdot \omega_0 \cdot C_k} \qquad C'_k = \frac{\Delta \cdot C_k}{\omega_0}$$

Summary of Prototype Filter Transformations

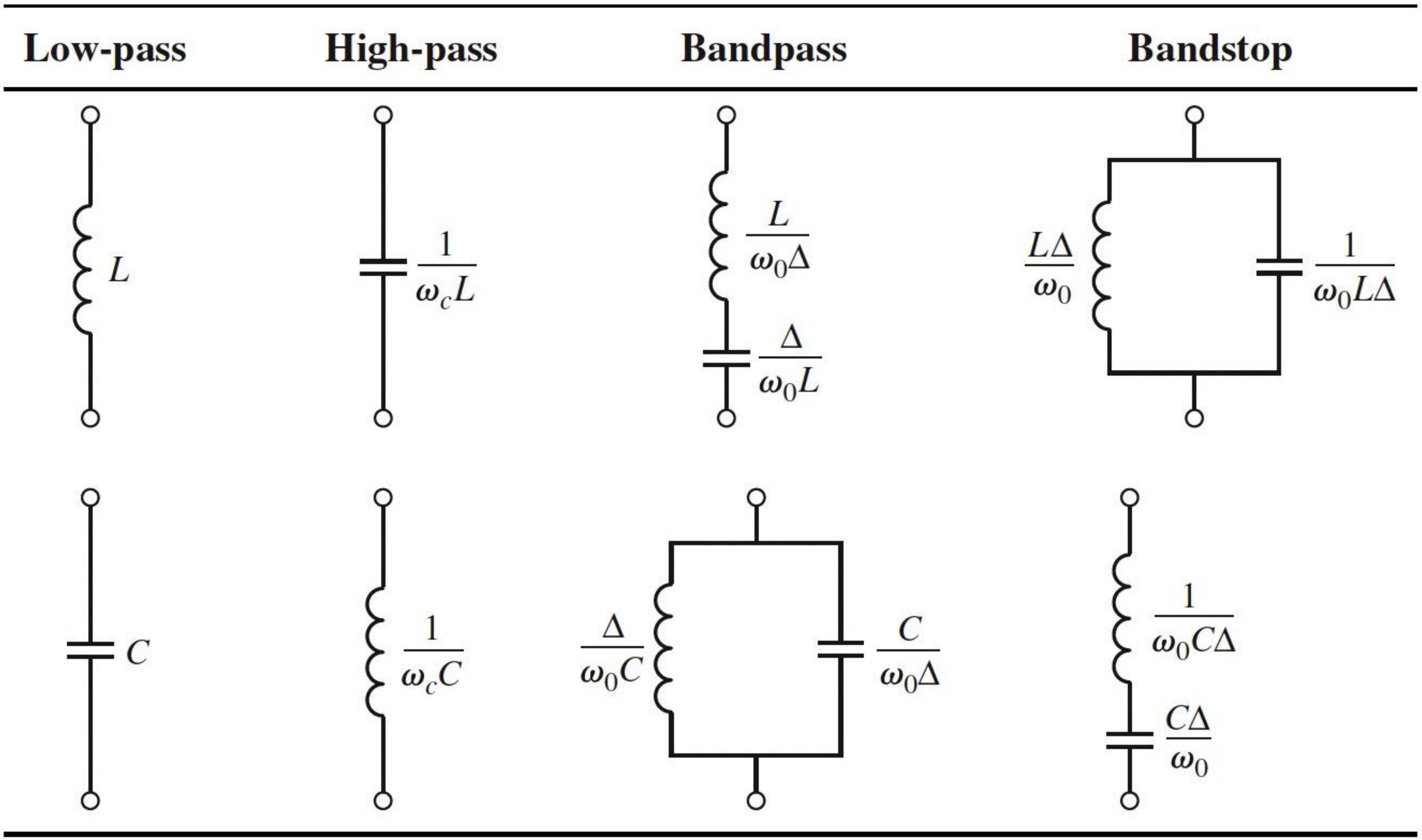


Table 8.6

Example

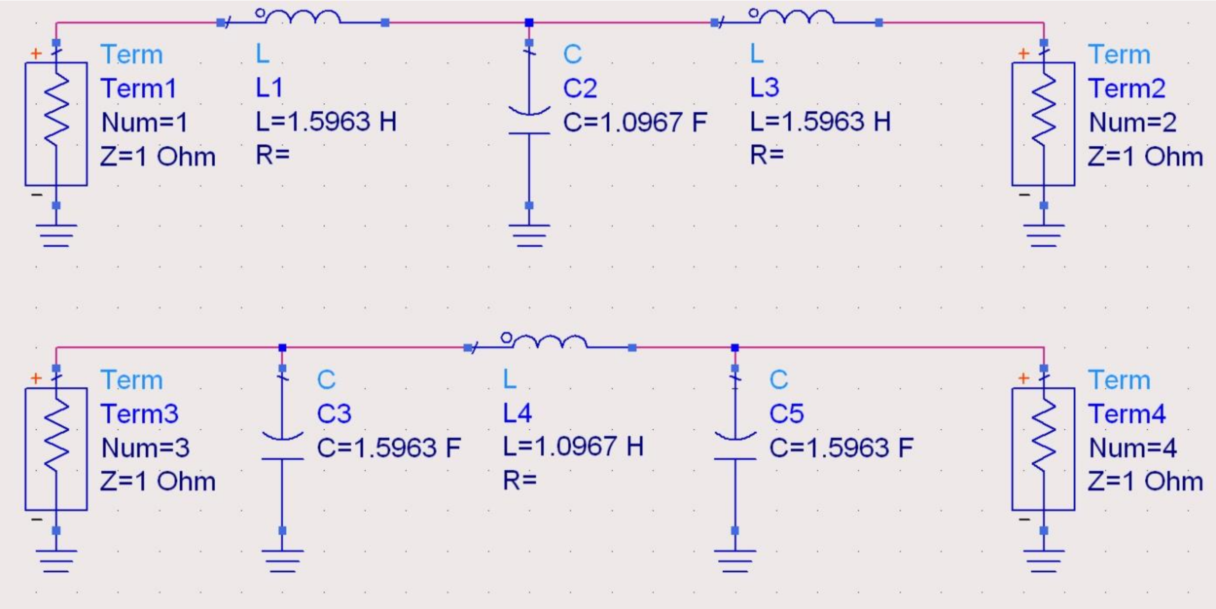
- Design a 3rd order **bandpass** filter with 0.5 dB ripples in passband. The **center frequency** of the filter should be 1 GHz. The **fractional bandwidth** of the passband should be 10%, and the **impedance** 50Ω.

$$\omega_0 = 2 \cdot \pi \cdot 1GHz = 6.283 \cdot 10^9 rad/s$$

$$\Delta = 0.1$$

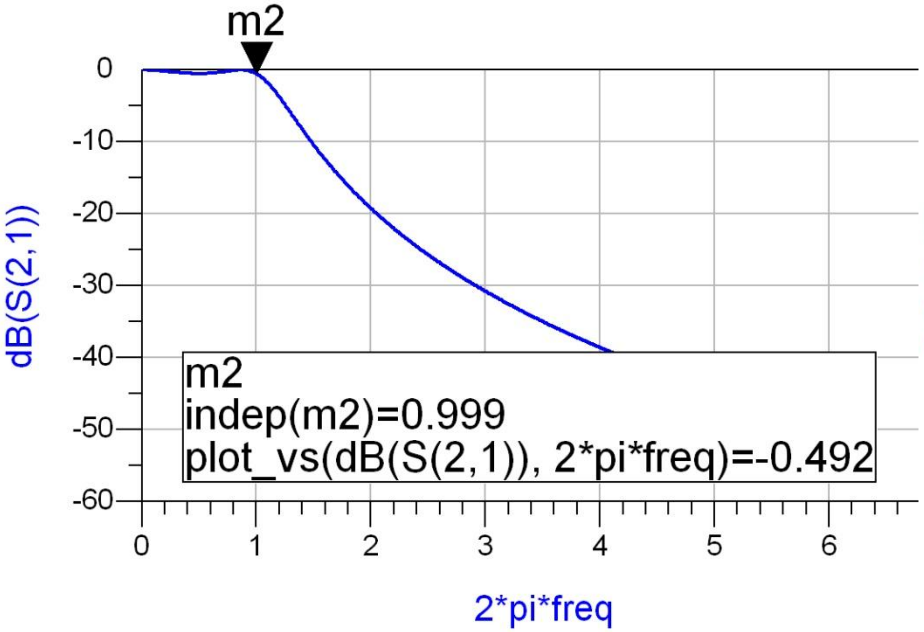
LPF Prototype

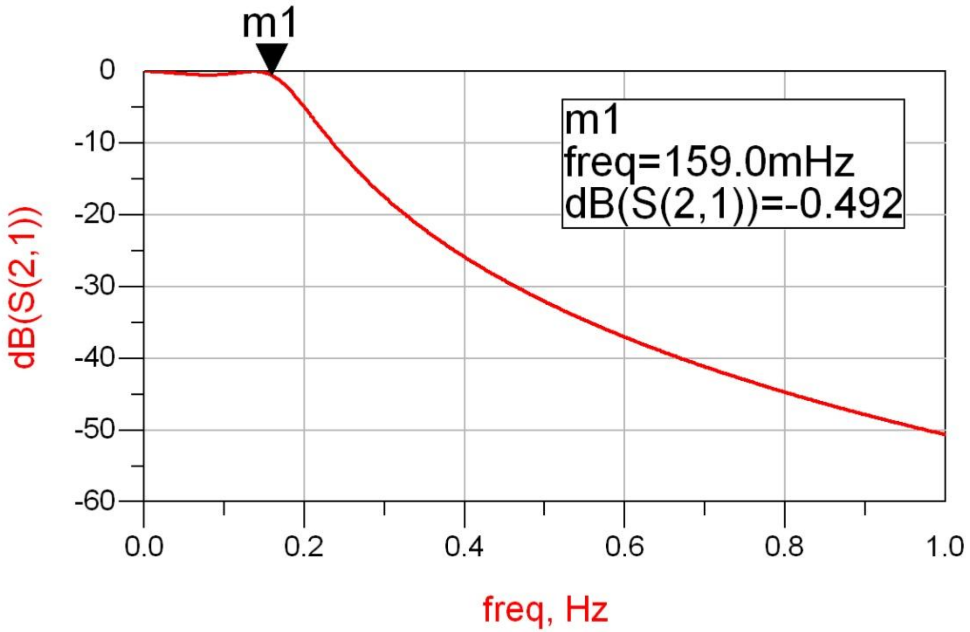
- 0.5dB equal-ripple table or design formulas:
  - g1 = 1.5963 = L1/C3,
  - g2 = 1.0967 = C2/L4,
  - g3 = 1.5963 = L3/C5,
  - g4=1.000 =R_L



LPF Prototype

- ω_o = 1 rad/s ($f_o = \omega_o / 2\pi$ = 0.159 Hz)





Bandpass Transformation / BPF

$$\omega_0 = 2 \cdot \pi \cdot 1GHz = 6.283 \cdot 10^9 \, rad/s \qquad \Delta = \frac{\Delta\omega}{\omega_0} = \frac{\Delta f}{f_0} = 0.1 \qquad R_0 = 50\,\Omega$$

g1 = 1.5963 = L1,
g2 = 1.0967 = C2,

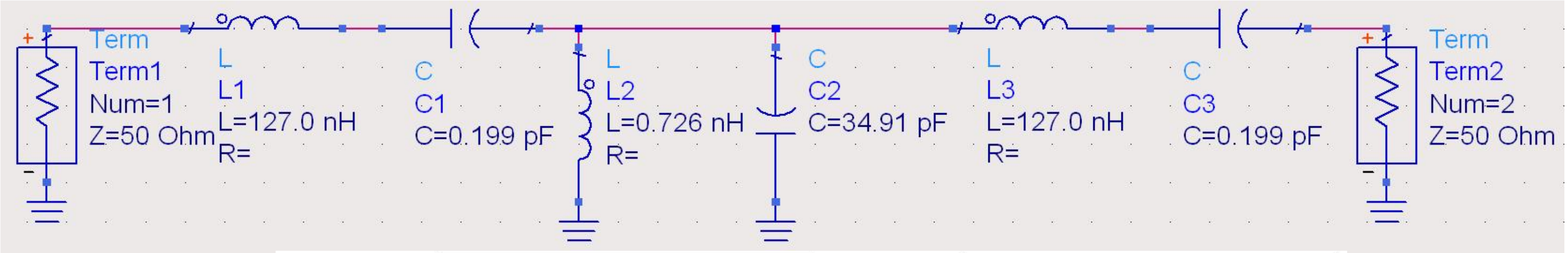
g3 = 1.5963 = L3,
g4=1.000 =R_L

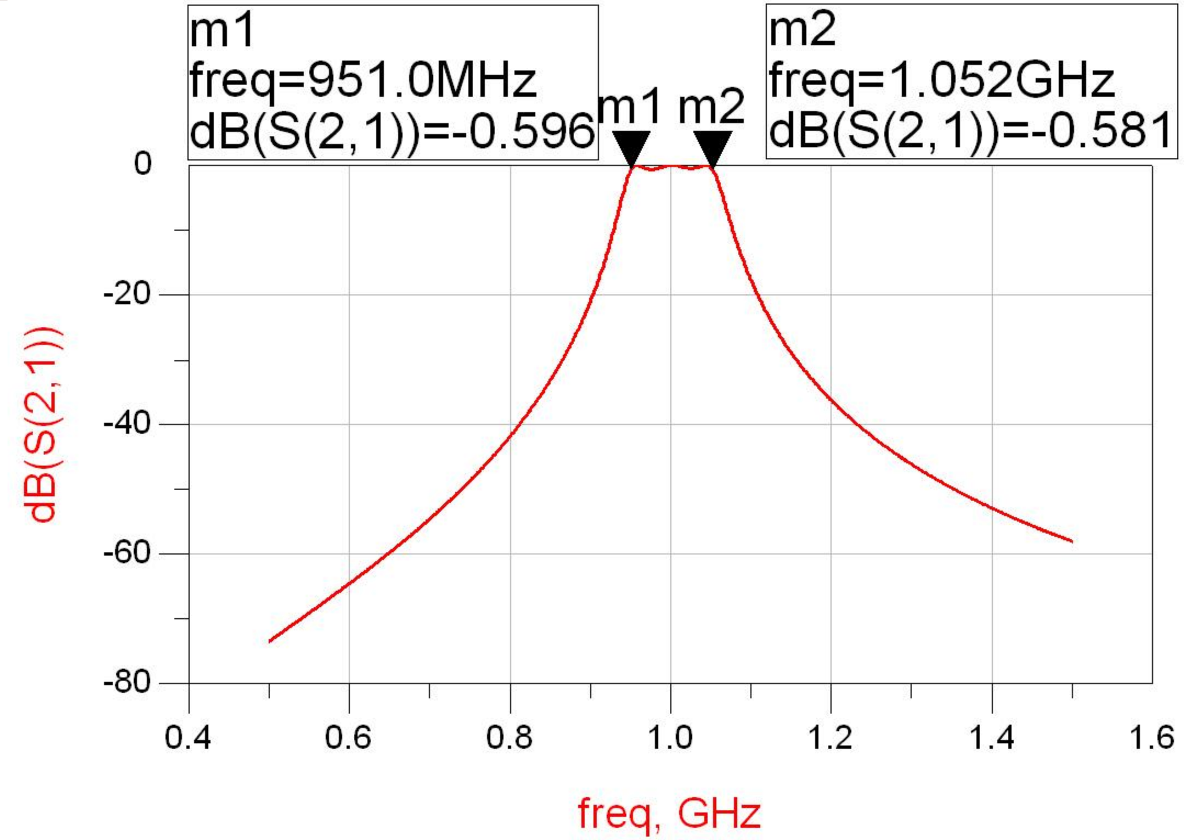
$$L_1' = \frac{L_1 \cdot R_0}{\Delta \cdot \omega_0} = 127.0\,nH \qquad C_1' = \frac{\Delta}{\omega_0 \cdot L_1 \cdot R_0} = 0.199\,pF$$

$$L_2' = \frac{\Delta \cdot R_0}{\omega_0 \cdot C_2} = 0.726\,nH \qquad C_2' = \frac{C_2}{\Delta \cdot \omega_0 \cdot R_0} = 34.91\,pF$$

$$L_3' = \frac{L_3 \cdot R_0}{\Delta \cdot \omega_0} = 127.0\,nH \qquad C_3' = \frac{\Delta}{\omega_0 \cdot L_3 \cdot R_0} = 0.199\,pF$$

ADS


Term
Term1
Num=1
Z=50 Ohm
L
L1
L=127.0 nH
R=
C
C1
C=0.199 pF
L
L2
L=0.726 nH
R=
C
C2
C=34.91 pF
L
L3
L=127.0 nH
R=
C
C3
C=0.199 pF
Term
Term2
Num=2
Z=50 Ohm


m1
freq=951.0MHz
dB(S(2,1))=-0.596
m2
freq=1.052GHz
dB(S(2,1))=-0.581
m1
m2
0
-20
-40
-60
-80
dB(S(2,1))
0.4
0.6
0.8
1.0
1.2
1.4
1.6
freq, GHz

Microwave Filters Implementation

Microwave Filters Implementation

- The lumped-element (L, C) filter design generally works well **only** at low frequencies (RF):
  - lumped-element inductors and capacitors are generally available only for a limited range of values, and can be difficult to implement at microwave frequencies
  - difficulty to obtain the (very low) required tolerance for elements

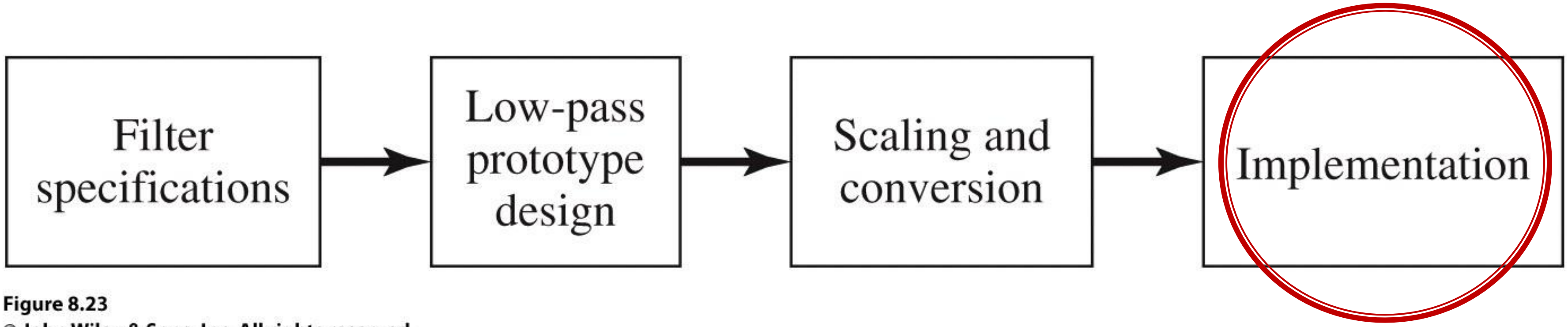


Figure 8.23

Richards' Transformation

- Impedance seen at the input of a line loaded with Z_L

$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

- We prefer the load impedance to be:
  - open circuit ($Z_L = \infty$) $\quad Z_{in,oc} = -j \cdot Z_0 \cdot \cot \beta \cdot l$
  - short circuit ($Z_L = 0$) $\quad Z_{in,sc} = j \cdot Z_0 \cdot \tan \beta \cdot l$
- Input impedance is:
  - capacitive $\quad Z_{in,oc} = j \cdot X_C = \frac{1}{j \cdot B_C} \qquad Z_0 \leftrightarrow \frac{1}{C} \quad \tan \beta \cdot l \leftrightarrow \omega$
  - inductive $\quad Z_{in,sc} = j \cdot X_L \qquad Z_0 \leftrightarrow L \quad \tan \beta \cdot l \leftrightarrow \omega$

Richards' Transformation

- Variable substitution

$$\Omega = \tan \beta \cdot l = \tan \left(\frac{\omega \cdot l}{v_p} \right)$$

- With this variable substitution we define:
  - reactance of an inductor

$$j \cdot X_L = j \cdot \Omega \cdot L = j \cdot L \cdot \tan \beta \cdot l$$

  - susceptance of a capacitor

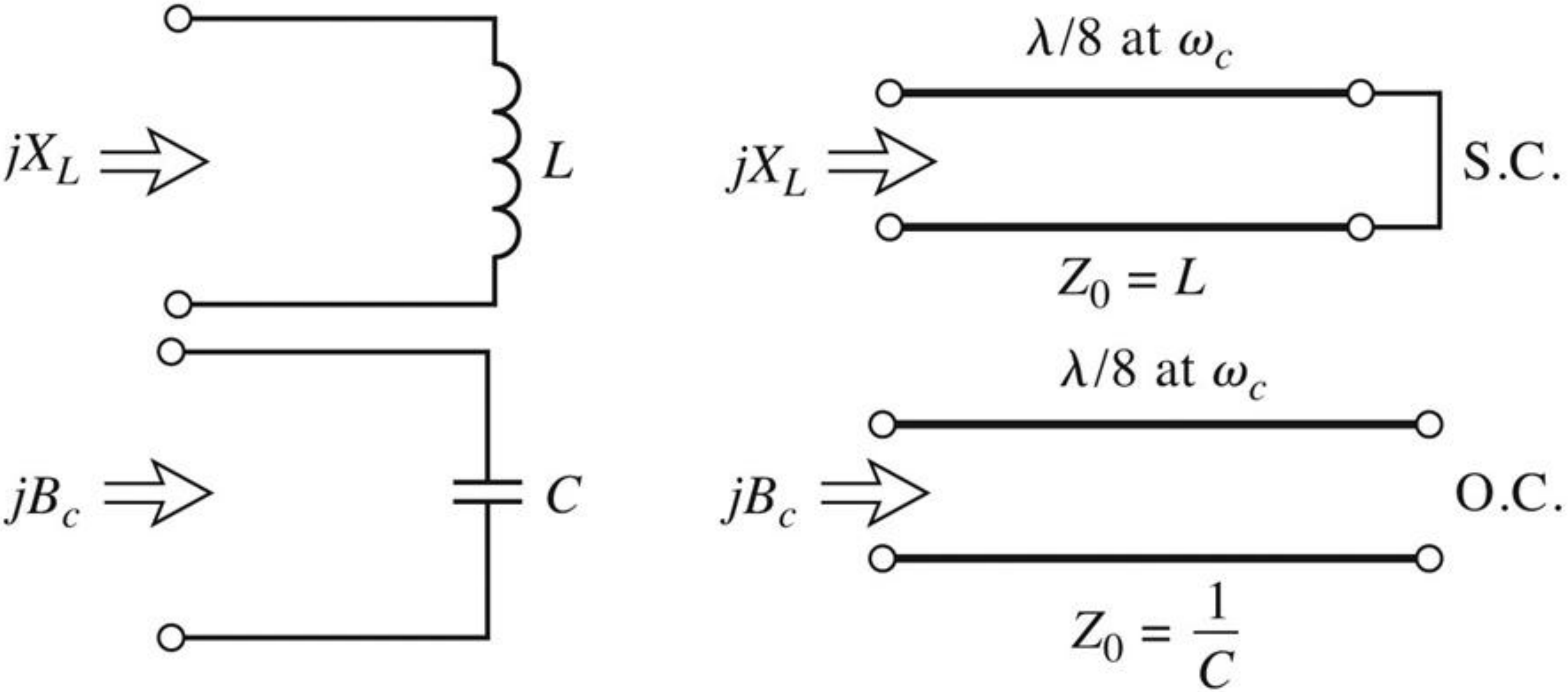
$$j \cdot B_C = j \cdot \Omega \cdot C = j \cdot C \cdot \tan \beta \cdot l$$

- The equivalent filter in Ω has a cutoff frequency at:

$$\Omega = 1 = \tan \beta \cdot l \quad \rightarrow \quad \beta \cdot l = \frac{\pi}{4} \quad \rightarrow \quad l = \frac{\lambda}{8}$$

Richards' Transformation

- allows implementation of the inductors and capacitors with lines **after** the transformation of the LPF prototype to the required type (LPF/HPF/BPF/BSF)



Richards' Transformation

- By choosing the open-circuited or short-circuited lines to be λ/8 at the desired cutoff frequency (ω_c) and the corresponding characteristic impedances (L/C from LPF prototype) we will obtain at frequencies around ω_c a behavior similar to that of the prototype filter.
  - At frequencies far from ω_c the behavior of the filter will no longer be identical to that of the prototype (in specific situations the correct behavior must be **verified**)
  - Frequency scaling is simplified: choosing the appropriate physical length of the line to have the electrical length λ/8 at the desired cutoff frequency
- All lines will have equal electrical lengths (λ/8) and thus comparable physical lengths, so the lines are called **commensurate** lines

Richards' Transformation

- At the frequency $\omega=2\cdot\omega_c$ the lines will be $\lambda/4$ long

$$l = \frac{\lambda}{4} \quad \Rightarrow \quad \beta \cdot l = \frac{\pi}{2} \quad \Rightarrow \quad \tan \beta \cdot l \to \infty$$

- an supplemental attenuation pole will occur at $2\cdot\omega_c$ (LPF):
  - inductances (usually in series) $Z_{in,sc} = j \cdot Z_0 \cdot \tan \beta \cdot l \to \infty$
  - capacitances (usually shunt) $Z_{in,oc} = -j \cdot Z_0 \cdot \cot \beta \cdot l \to 0$

Richards' Transformation

- the periodicity of tan function implies the periodicity of the filter implemented with lines
  - the filter response will be repeated every $4 \cdot \omega_c$

$$\tan(\alpha + \pi) = \tan \alpha$$

$$\beta \cdot l\Big|_{\omega=\omega_c} = \frac{\pi}{4} \quad \Rightarrow \quad \frac{\omega_c \cdot l}{v_p} = \frac{\pi}{4} \quad \Rightarrow \quad \pi = \frac{(4 \cdot \omega_c) \cdot l}{v_p}$$

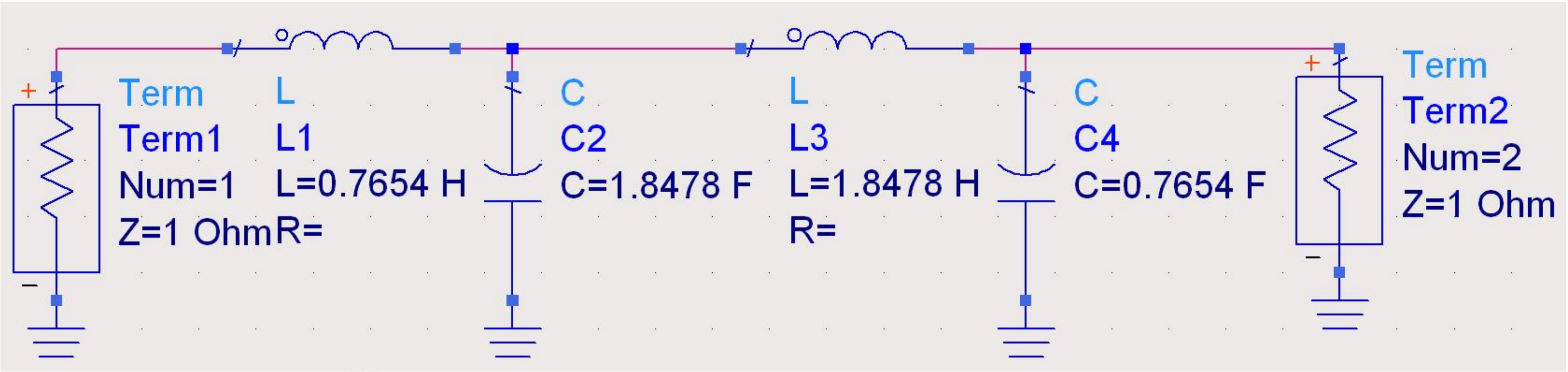
$$Z_{in}(\omega) = Z_{in}(\omega + 4 \cdot \omega_c) \quad \Rightarrow \quad P_{LR}(\omega) = P_{LR}(\omega + 4 \cdot \omega_c)$$

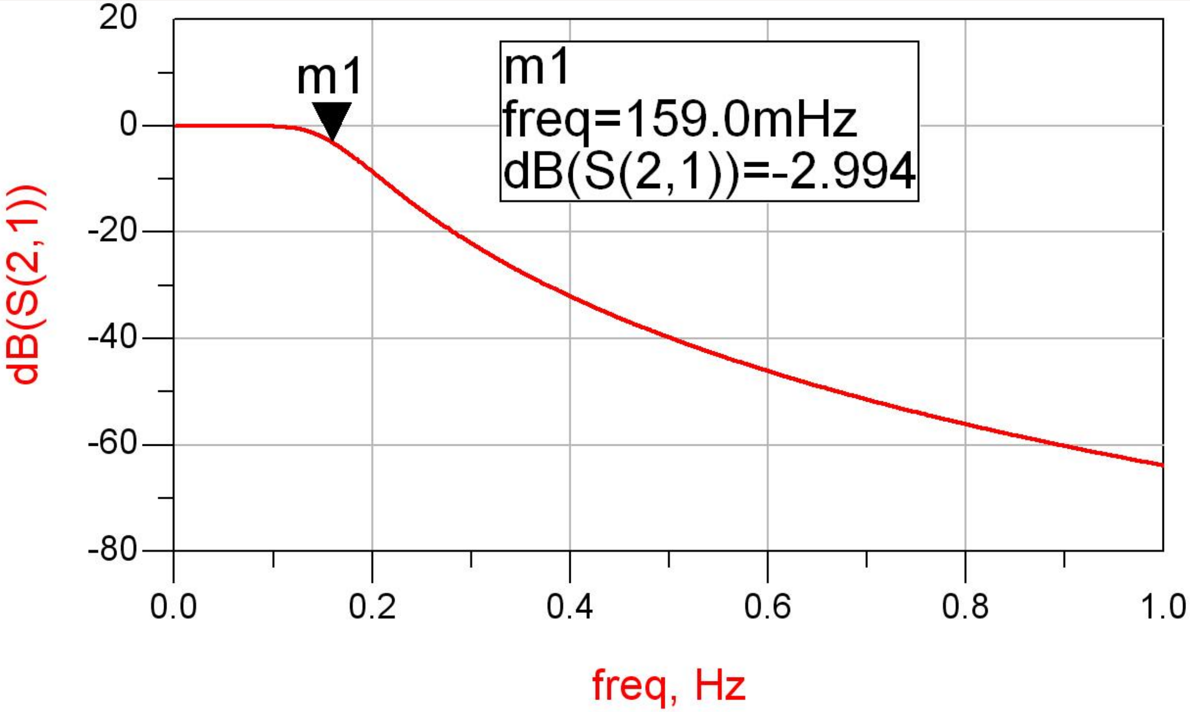
$$P_{LR}(4 \cdot \omega_c) = P_{LR}(0) \quad P_{LR}(3 \cdot \omega_c) = P_{LR}(-\omega_c) \quad P_{LR}(5 \cdot \omega_c) = P_{LR}(\omega_c)$$

Example

- Low-pass filter 4^{th} order, 4 GHz cutoff frequency, maximally flat design (working with 50Ω source and load)
- maximally flat table or formulas:
  - $g_1 = 0.7654 = L_1$
  - $g_2 = 1.8478 = C_2$
  - $g_3 = 1.8478 = L_3$
  - $g_4 = 0.7654 = C_4$
  - $g_5 = 1$ (**does not** need supplemental impedance matching – required only for even order equal-ripple filters)

LPF Prototype





Lumped elements

$$\omega_c = 2 \cdot \pi \cdot 4GHz = 2.5133 \cdot 10^{10} rad/s$$

g1 = 0.7654 = L1,
g2 = 1.8478 = C2,
g3 = 1.8478 = L3,
g4 = 0.7654 = C4,
g5 = 1 = RL

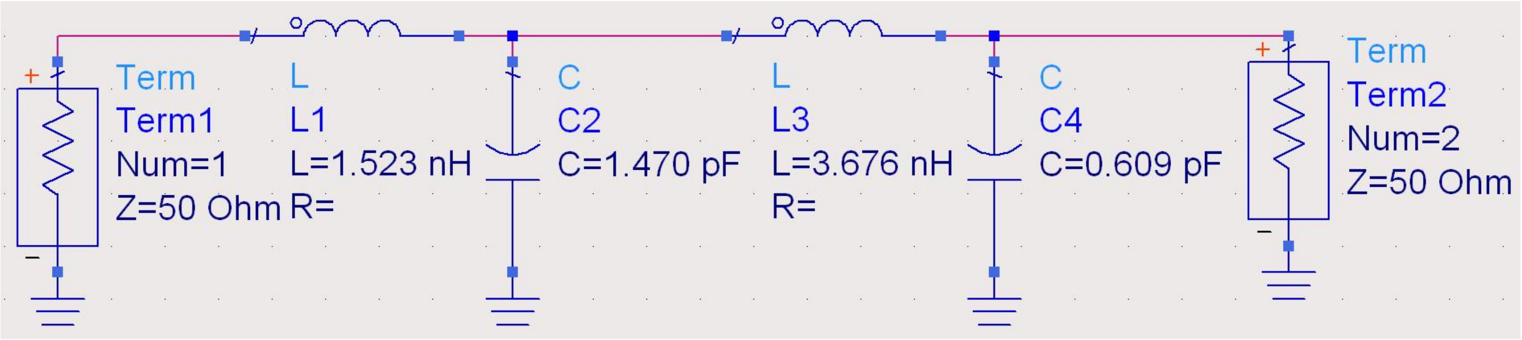
$$L'_1 = \frac{R_0 \cdot L_1}{\omega_c} = 1.523nH$$

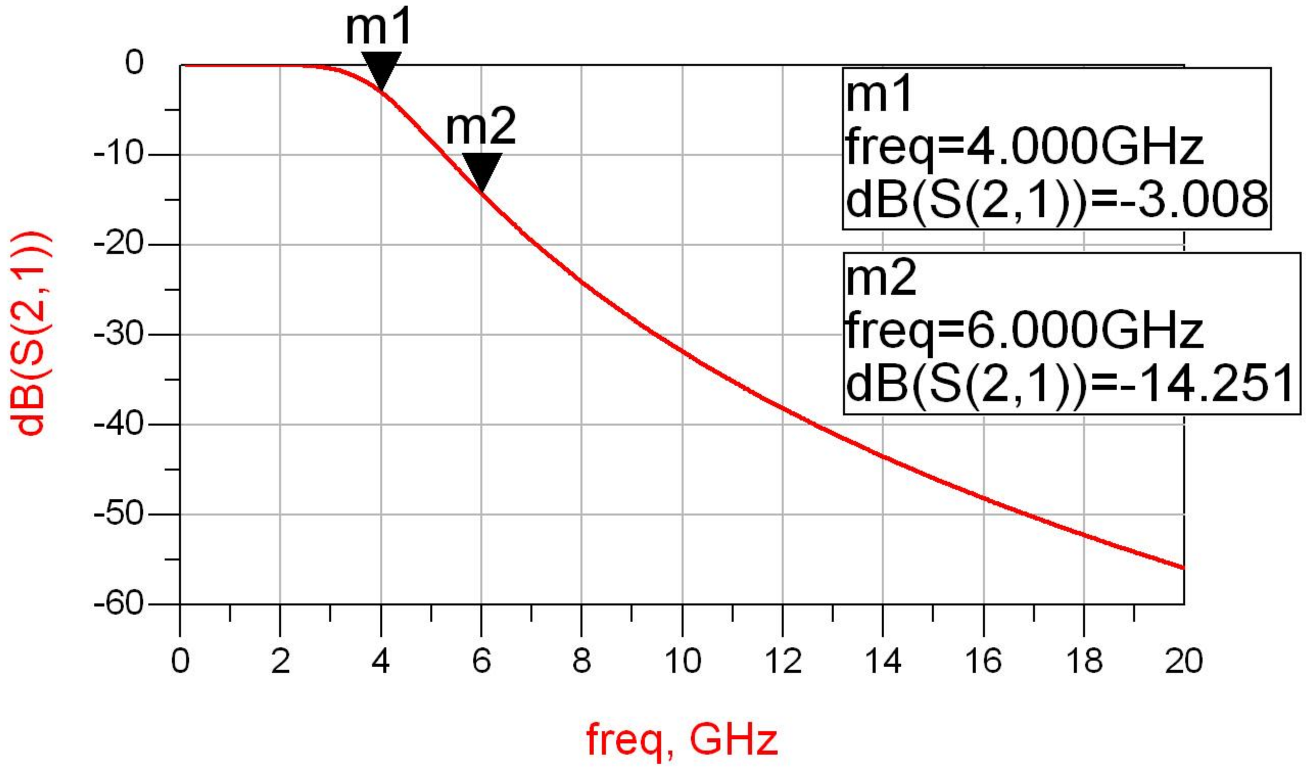
$$C'_2 = \frac{C_2}{R_0 \cdot \omega_c} = 1.470pF$$

$$L'_3 = \frac{R_0 \cdot L_3}{\omega_c} = 3.676nH$$

$$C'_4 = \frac{C_4}{R_0 \cdot \omega_c} = 0.609pF$$

Lumped elements – ADS





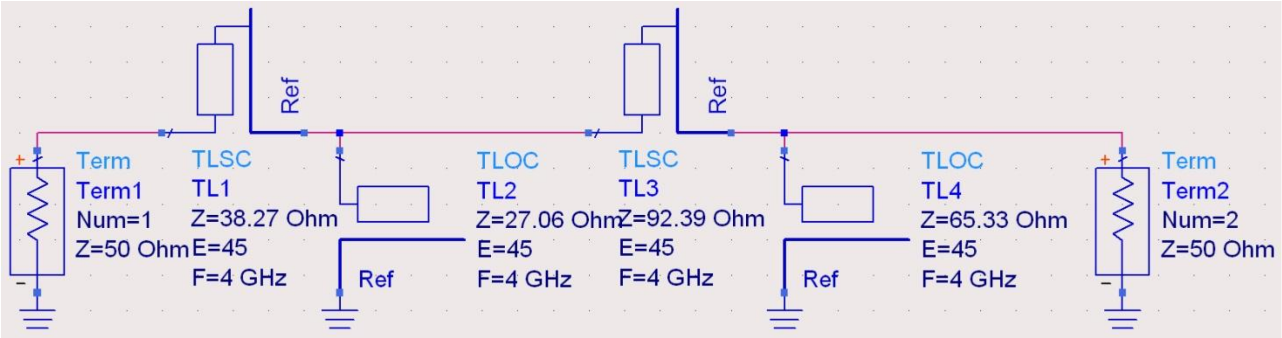
Richards' Transformation

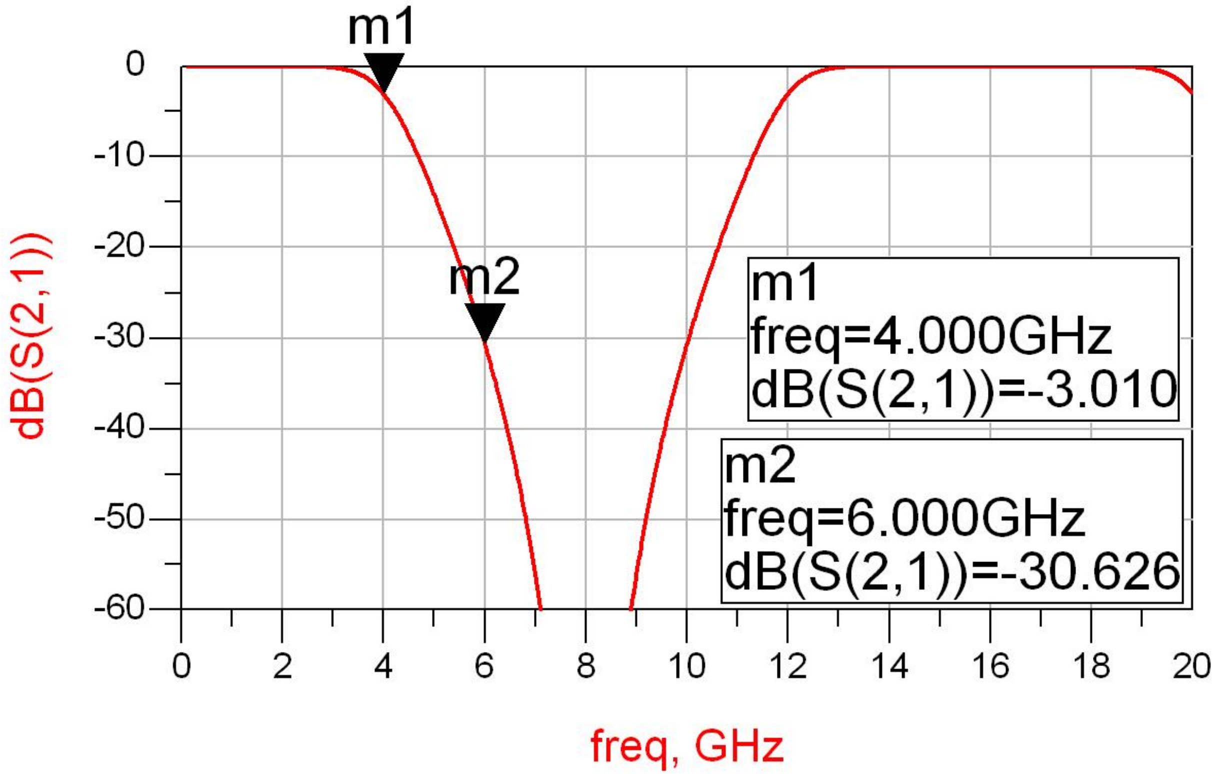
- LPF Prototype parameters:
  - g1 = 0.7654 = L1
  - g2 = 1.8478 = C2
  - g3 = 1.8478 = L3
  - g4 = 0.7654 = C4
- Normalized line impedances
  - z1 = 0.7654 = series / short circuit
  - z2 = 1 / 1.8478 = 0.5412 = shunt / open circuit
  - z3 = 1.8478 = series / short circuit
  - z4 = 1/ 0.7654 = 1.3065 = shunt / open circuit
- Impedance scaling by multiplying with Zo = 50Ω
- All lines must have the length equal to λ/8 (electrical length E = 45°) at 4GHz

$$Z_0 \leftrightarrow \frac{1}{C}$$

$$Z_0 \leftrightarrow L$$

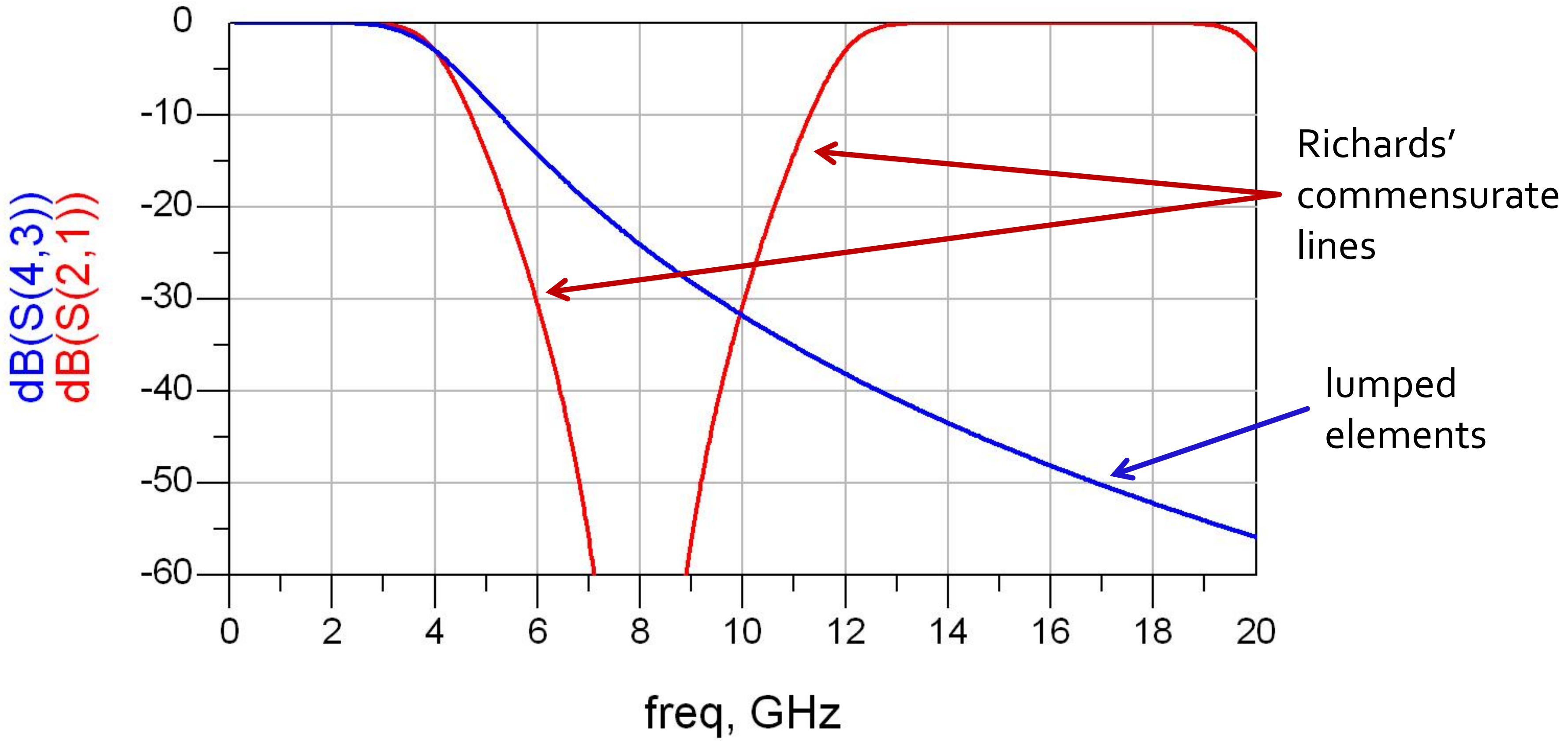
Richards' Transformation – ADS





Richards' Transformation

- Filters implemented with Richards' Transformation
  - beneficiate from the supplemental pole at $2 \cdot \omega_c$
  - have the major disadvantage of frequency periodicity, a supplemental non-periodic LPF must be inserted if needed



Equal-ripple prototype

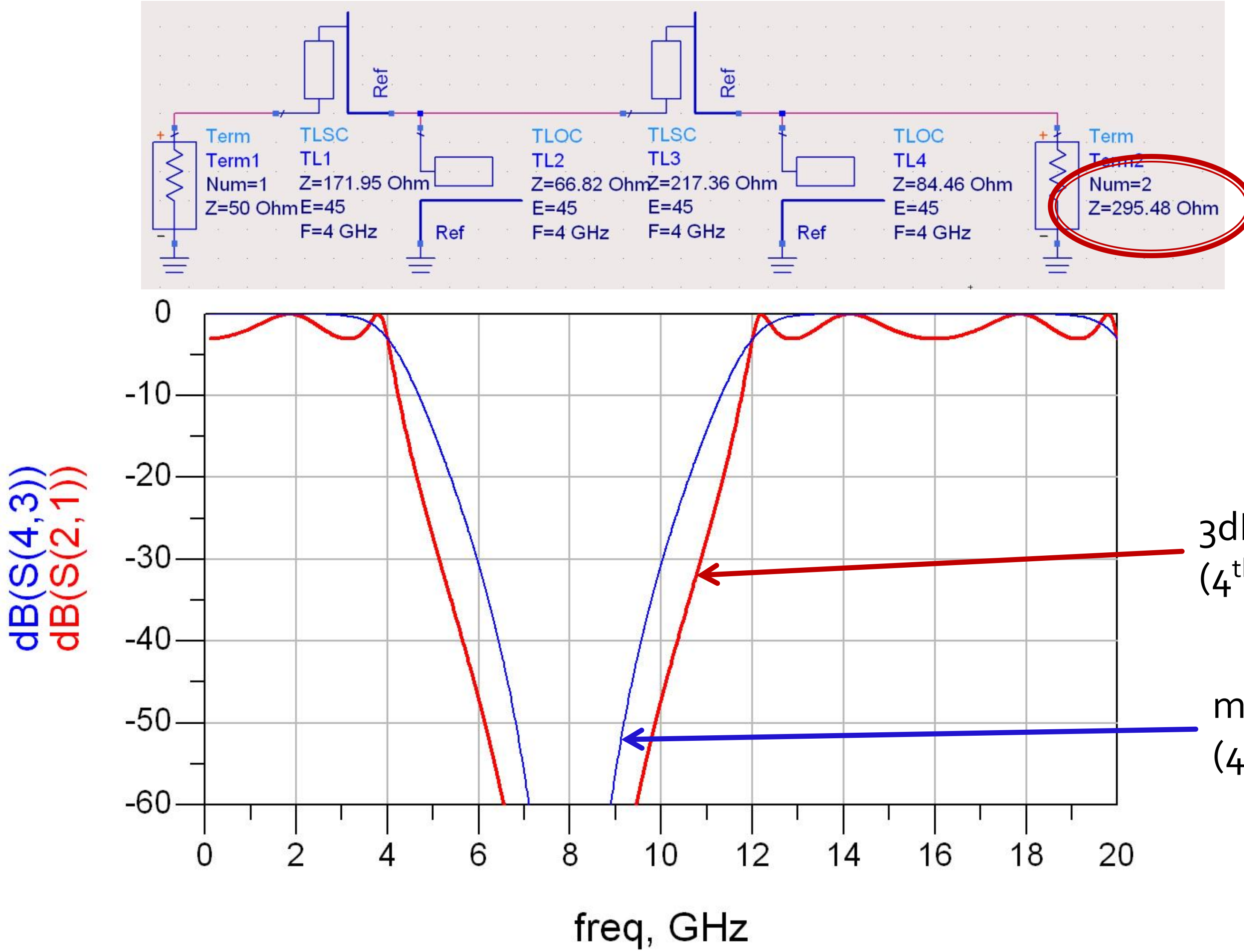
- For even N order of the filter (N = 2, 4, 6, 8 …) equal-ripple filters **must** closed by a non-standard load impedance $\mathbf{g_{N+1} \neq 1}$
- If the application doesn't allow this, supplemental impedance matching is required (quarter-wave transformer, binomial …) to $\mathbf{g_L = 1}$

$$g_{N+1} \neq 1 \rightarrow R \neq R_0 \quad (50\Omega)$$

Observation: even order equal-ripple

- Same filter, 3dB equal-ripple
- 3dB equal-ripple tables or formulas:
  - $g1 = 3.4389 = L1$
  - $g2 = 0.7483 = C2$
  - $g3 = 4.3471 = L3$
  - $g4 = 0.5920 = C4$
  - $g5 = 5.8095 = R_L$
- Line impedances
  - $Z1 = 3.4389 \cdot 50\Omega = 171.945\Omega$ = series / short circuit
  - $Z2 = 50\Omega / 0.7483 = 66.818\Omega$ = shunt / open circuit
  - $Z3 = 4.3471 \cdot 50\Omega = 217.355\Omega$ = series / short circuit
  - $Z4 = 50\Omega / 0.5920 = 84.459\Omega$ = shunt / open circuit
  - $RL = 5.8095 \cdot 50\Omega = 295.475\Omega$ = load

Even order equal-ripple – ADS

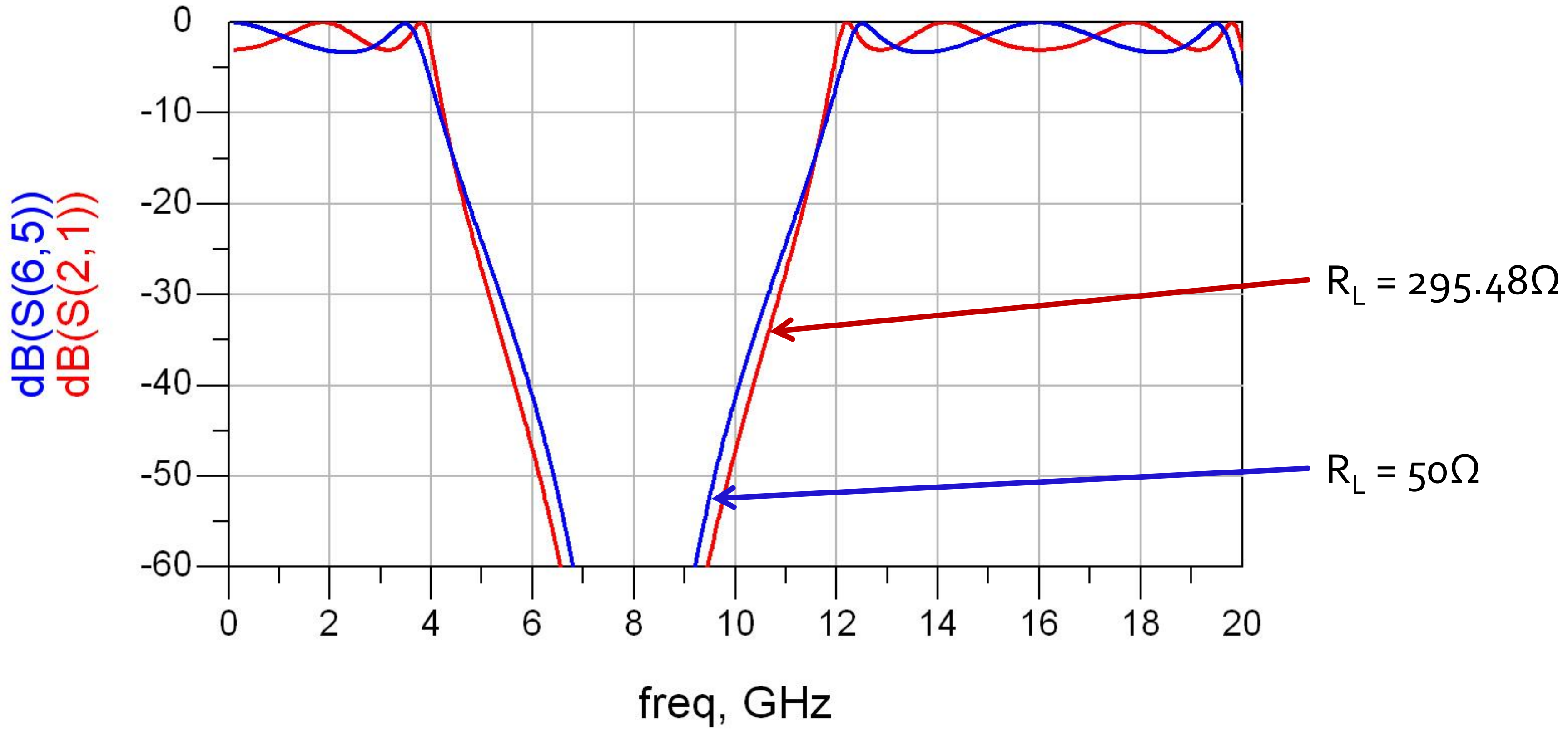


3dB equal-ripple (4th ord.)

maximum flat (4th ord)

Observation: even order equal-ripple

- Even order equal-ripple filters need output matching towards 50Ω for precise results. Example:



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